



North Carolina Department of Public Instruction

INSTRUCTIONAL SUPPORT TOOLS

FOR ACHIEVING NEW STANDARDS

8th Grade Mathematics • Unpacked Contents

For the new Standard Course of Study that will be effective in all North Carolina schools in the 2018-19 School Year.

This document is designed to help North Carolina educators teach the 8th Grade Mathematics Standard Course of Study. NCDPI staff are continually updating and improving these tools to better serve teachers and districts.

What is the purpose of this document?

The purpose of this document is to increase student achievement by ensuring educators understand the expectations of the new standards. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the NC SCOS.

What is in the document?

This document includes a detailed clarification of each standard in the grade level along with a *sample* of questions or directions that may be used during the instructional sequence to determine whether students are meeting the learning objective outlined by the standard. These items are included to support classroom instruction and are not intended to reflect summative assessment items. The examples included may not fully address the scope of the standard. The document also includes a table of contents of the standards organized by domain with hyperlinks to assist in navigating the electronic version of this instructional support tool.

How do I send Feedback?

Link for: [Feedback for NC's Math Unpacking Documents](#) We will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?

Link for: [NC Mathematics Standards](#)

North Carolina 8th Grade Standards

Standards for Mathematical Practice

The Number System	Expressions & Equations	Functions	Geometry	Statistics & Probability
<p>Know that there are numbers that are not rational and approximate them by rational numbers.</p> <p>NC.8.NS.1 NC.8.NS.2</p>	<p>Work with radicals and integer exponents.</p> <p>NC.8.EE.1 NC.8.EE.2 NC.8.EE.3 NC.8.EE.4</p> <p>Analyze and solve linear equations and inequalities.</p> <p>NC.8.EE.7</p> <p>Analyze and solve pairs of simultaneous linear equations.</p> <p>NC.8.EE.8</p>	<p>Define, evaluate, and compare functions.</p> <p>NC.8.F.1 NC.8.F.2 NC.8.F.3</p> <p>Use functions to model relationships between quantities.</p> <p>NC.8.F.4 NC.8.F.5</p>	<p>Understand congruence and similarity using physical models, transparencies, or geometry software.</p> <p>NC.8.G.2 NC.8.G.3 NC.8.G.4</p> <p>Analyze angle relationships.</p> <p>NC.8.G.5</p> <p>Understand and apply the Pythagorean Theorem.</p> <p>NC.8.G.6 NC.8.G.7 NC.8.G.8</p> <p>Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</p> <p>NC.8.G.9</p>	<p>Investigate patterns of association in bivariate data.</p> <p>NC.8.SP.1 NC.8.SP.2 NC.8.SP.3 NC.8.SP.4</p>

Standards for Mathematical Practice

Practice	Explanations and Examples
1. Make sense of problems and persevere in solving them.	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
2. Reason abstractly and quantitatively.	In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree to which the pattern models a line. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatter plots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
6. Attend to precision.	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
8. Look for and express regularity in repeated reasoning.	In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. They analyze patterns of repeating decimals to identify the corresponding fraction. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Return to: [Standards](#)

The Number System

Know that there are numbers that are not rational and approximate them by rational numbers.
NC.8.NS.1 Understand that every number has a decimal expansion. Building upon the definition of a rational number, know that an irrational number is defined as a non-repeating, non-terminating decimal.

Clarification	Checking for Understanding
<p>In 6th grade students were introduced to integers and rational numbers. In 7th grade, students formalized the definition of rational numbers. Students build on this knowledge to complete their understanding of the Real Number System by recognizing irrational numbers and their relationship to rational numbers. Students understand an irrational number, when represented as a decimal, is non-repeating and non-terminating and irrational numbers cannot be written as a rational number. It is important for students to understand that distinction between fractional form and a rational number, as irrational numbers are often written in fractional form. For example, $\frac{3\pi}{4}$, is an irrational number written in fractional form. Students are able to identify irrational numbers.</p>	<p>Create a graphic organizer to show the relationships within the real number system, including natural numbers, whole numbers, integers, rational numbers and irrational numbers. Include examples that are exclusively within each type of number.</p>

Return to: [Standards](#)

Know that there are numbers that are not rational and approximate them by rational numbers.
NC.8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers and locate them approximately on a number line. Estimate the value of expressions involving:

- Square roots and cube roots to the tenths.
- π to the hundredths.

Clarification	Checking for Understanding								
<p>Students estimate the value of an irrational number and use that estimate to compare an irrational number to other numbers and to place irrational numbers on a number line. Students estimate expressions containing square roots and cube roots to at least the tenths place. Students should estimate expressions containing π to the hundredths place. This standard connects strongly to NC.8.EE.1, where students write the solutions to equations of the form $x^2 = p$ and $x^3 = p$ as square or cubed roots.</p>	<p>Graph the following on a number line: $\frac{3}{2}, \sqrt{2}, 1.\bar{3}, -\frac{7}{8}, -\frac{\sqrt{3}}{2}$</p> <hr/> <p>Estimate the following expressions to the tenths.</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">a) $10\sqrt{3}$</td> <td style="width: 50%;">b) $-\frac{\sqrt{54}}{2}$</td> </tr> <tr> <td>c) $\sqrt[3]{218}$</td> <td>d) $\sqrt{75}$</td> </tr> </table> <hr/> <p>Estimate the following expressions to the hundredths.</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">a) 12π</td> <td style="width: 50%;">b) $4\pi + 10$</td> </tr> <tr> <td>c) $\frac{3\pi}{2}$</td> <td>d) $16 - 4\pi$</td> </tr> </table>	a) $10\sqrt{3}$	b) $-\frac{\sqrt{54}}{2}$	c) $\sqrt[3]{218}$	d) $\sqrt{75}$	a) 12π	b) $4\pi + 10$	c) $\frac{3\pi}{2}$	d) $16 - 4\pi$
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Return to: [Standards](#)

Expressions and Equations

Work with radicals and integer exponents. NC.8.EE.1 Develop and apply the properties of integer exponents to generate equivalent numerical expressions.											
Clarification Students first worked with whole number exponents in 6 th grade. At this grade level, students will build upon that knowledge to understand the properties of integer exponents and numerical bases and patterns of repeated multiplication and division. Students use their understanding of exponents as repeated multiplication to develop and create equivalent expressions and justify the following properties: <ul style="list-style-type: none"> • $5^3 \cdot 5^4 = 5^{3+4} = 5^7$ • $\frac{5^3}{5^4} = 5^{3-4} = 5^{-1} = \frac{1}{5}$ • $(5^3)^4 = 5^{3 \cdot 4} = 5^{12}$ • $5^3 \cdot 2^3 = (5 \cdot 2)^3 = (10)^3$ • $5^0 = 1$ • $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ Eighth grade is the first time students raise negative numbers to a power. Students recognize that negative numbers raised to an even power produce different products when parentheses are used. For example, -4^2 and $(-4)^2$ have products of -16 and 16 respectively. Students are not expected to know the names of the properties of exponents.	Checking for Understanding Rewrite the following expressions so that each expression does not contain an exponent. <table style="width: 100%; margin-top: 10px;"> <tr> <td style="width: 50%;">a) $\frac{2^3}{5^2}$</td> <td style="width: 50%;">b) $\frac{2^2}{2^6}$</td> </tr> <tr> <td>c) 6^0</td> <td>d) $\frac{3^{-2}}{2^4}$</td> </tr> <tr> <td>e) $(3^2)(3^4)$</td> <td>f) $(4^3)^2$</td> </tr> <tr> <td>g) $-\left(\frac{2}{3}\right)^2$</td> <td>h) $\left(-\frac{5}{4}\right)^2 \cdot \left(\frac{2}{5}\right)^{-3}$</td> </tr> <tr> <td>i) $\frac{(3^2)^4}{(3^2)(3^3)}$</td> <td>j) $12^7 \cdot 12^{-7}$</td> </tr> </table>	a) $\frac{2^3}{5^2}$	b) $\frac{2^2}{2^6}$	c) 6^0	d) $\frac{3^{-2}}{2^4}$	e) $(3^2)(3^4)$	f) $(4^3)^2$	g) $-\left(\frac{2}{3}\right)^2$	h) $\left(-\frac{5}{4}\right)^2 \cdot \left(\frac{2}{5}\right)^{-3}$	i) $\frac{(3^2)^4}{(3^2)(3^3)}$	j) $12^7 \cdot 12^{-7}$
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Return to: [Standards](#)

Work with radicals and integer exponents. NC.8.EE.2 Use square root and cube root symbols to: <ul style="list-style-type: none"> • Represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. • Evaluate square roots of perfect squares and cube roots of perfect cubes for positive numbers less than or equal to 400. 					
Clarification This standard introduces the inverse relationship between squares and square roots and cubes and cube roots. Students should understand that: <ul style="list-style-type: none"> • \sqrt{p} is defined as the positive solution to $x^2 = p$. • $\sqrt[3]{p}$ is defined as the solution to $x^3 = p$. Represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. In 8 th grade, p is limited to be a positive rational number. In NC.8.NS.2, students learn to estimate a square root or cube root to the tenths place. Evaluate square roots of perfect squares and cube roots of perfect cubes for positive numbers less than or equal to 400. Student should know the definition of a perfect square and that: <ul style="list-style-type: none"> • $\sqrt{a^2} = \pm a$ • $\sqrt[3]{a^3} = a$ This is the first instance students will have seen an equation with potentially 2 solutions.	Checking for Understanding List all of the possible values for x in the following equation: $x^3 = 216$ <table style="width: 100%; margin-top: 5px;"> <tr> <td style="width: 50%;">A. $\sqrt{216}$</td> <td style="width: 50%;">B. $\sqrt[3]{216}$</td> </tr> <tr> <td>C. 6</td> <td>D. 72</td> </tr> </table> <hr/> Evaluate the following expression: $\sqrt{196}$ <hr/> Solve for r in the following equation: $r^2 = 81$	A. $\sqrt{216}$	B. $\sqrt[3]{216}$	C. 6	D. 72
A. $\sqrt{216}$	B. $\sqrt[3]{216}$				
C. 6	D. 72				

Return to: [Standards](#)

Work with radicals and integer exponents.

NC.8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities and to express how many times as much one is than the other.

Clarification	Checking for Understanding
<p>Students use their knowledge of the base ten number system and exponents to rewrite numbers using scientific notation. Students interpret scientific notation generated when using technology. Students compare numbers written in scientific notation and express the multiplicative relationship between the numbers.</p> <p>For example: Which of the following represents a larger number?</p> <p>a) 1.5×10^9 b) 7.5×10^7</p> <p><i>Solution:</i> 1.5×10^9 is the larger number</p> <p>For your answer, how many times larger is your answer than the smaller number?</p> <p><i>Solution:</i> 1.5×10^9 is 20 times larger than 7.5×10^7</p> <p><i>Notice that 1.5 is .2 times 7.5. Looking at just the 10s, 10^9 is 100 times larger than 10^7.</i></p> <p>$(0.2)(100) = 20$</p> <p><i>Students should see more than just a division problem but should see the multiplicative relationships that are unique to scientific notation.</i></p>	<p>Write the following into scientific notation:</p> <p>a) The distance between the sun and the Earth is 93,000,000 miles. b) The distance between the sun and Neptune is 2,795,000,000 miles. c) A type of fairyfly is the smallest known flying insect and is only 0.0059 inches long. d) An average bacterium is about 0.00004 inches long.</p> <p>Use the information from above to answer the following questions:</p> <p>e) In astronomy, the distance between the sun and the Earth is known as 1 AU, or astronomical unit. Measured in AUs, what is the distance between the sun and Neptune? f) If average sized bacteria were placed in a straight line, how many of bacteria would be needed to equal the length of the smallest known flying insect, a fairyfly?</p>

Return to: [Standards](#)

Work with radicals and integer exponents.

NC.8.EE.4 Perform multiplication and division with numbers expressed in scientific notation to solve real-world problems, including problems where both decimal and scientific notation are used.

Clarification	Checking for Understanding
<p>Students use the laws of exponents to multiply and divide expressions containing numbers written in scientific and decimal notation to solve real-world problems.</p>	<p>Write the answer to the following in both scientific and decimal notation.</p> <p>a) Patrice works at a museum giving tours. Patrice would like to know how many words she speaks in a year giving tours at her job. The average person speaks about 150 words per minute. Patrice led tours that were 25 minutes long, 6 times per day. About how many words would Patrice have spoken in a year? b) Jensen is building a snow fort. Each block in the fort weighs about 1 kilogram. Jensen hopes to make about 40 blocks for the fort. If a snowflake weighs about 3×10^{-3} grams, approximately how many snowflakes will be in the fort?</p>

Return to: [Standards](#)

Analyze and solve linear equations and inequalities.**NC.8.EE.7** Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable.

- Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions.
- Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides.

Clarification

In 7th grade, students learned to solve multistep one-variable equations and inequalities, with the variable on one side. In 8th grade, students will build upon this understanding to solve one-variable equations and inequalities with the same variable on both sides.

Students recognize and explain when linear equations have one solution, infinitely many solutions, or no solution without completing the solving process.

Linear inequalities may have infinitely many solutions or no solution.

For example: $x + 5 > 2x + 8 - x$

This inequality has no solution as 5 is not greater than 8.

Students justify their answer with mathematical reasoning, including the use of the properties of equality.

Checking for Understanding

Determine the number of solutions for each of the following. If there is only one solution, determine the solution.

a) $\frac{1}{2}(2p + 9) = -p + 5$

c) $8 - 2(n + 3) = n + 7 - 3n$

b) $5 - 3q = 4 - \frac{2}{3}(4.5q - 1.5)$

d) $\frac{1}{5}g + \frac{2}{5} = 1\frac{1}{5}g - 2\left(4 + \frac{2}{5}\right)$

In the following equation, a and b represent integers.

$$2x + a = 5 - bx$$

What values of a and b would create an equation with just one solution?

What values of a and b would create an equation with no solutions?

What values of a and b would create an equation with infinitely many solutions?

Solve the following and graph the solutions on a number line:

a) $3x - 2 > 9 + 5x$

c) $\frac{2}{3}h + 9 < 8\left(\frac{1}{3}h - 2\right)$

b) $\frac{5+2y}{4} \geq \frac{y+3}{2}$

d) $\frac{1}{5}(13 - 20x) \leq -14 - 4x$

Two companies are competing for a contract to make the programs for the high school football games. Howie's Printing charges a \$19.99 fee for printing and \$0.25 for each program printed. Mint Print charges a \$29.99 fee for printing and \$0.10 for each program printed.

For what number of printed programs will Howie's Printing cost more than Mint Print?

a) Write and solve an inequality to describe this situation.

b) Describe what your solution means.

c) If you anticipate needing 75 programs for a football game, which company is the cheaper choice?

Return to: [Standards](#)

Analyze and solve pairs of simultaneous linear equations.

NC.8.EE.8 Analyze and solve a system of two linear equations in two variables in slope-intercept form.

- Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously.
- Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection.

Clarification

Students find the solution to a system of two linear equations by graphing. In 8th grade, the linear equations will be limited to slope-intercept form.

Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously.

Students can explain and demonstrate that the solution to the system of equations must be a solution to each of the equations of the system. By comparing the graphs of a system to the corresponding equations in the system, students can discover characteristics of systems that have no solutions, one solution, and infinite solutions.

Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection.

Students write a system of equations to represent a word problem. Word problems will be presented in a way that students can write the equation directly into slope-intercept form. Students recognize the structure of equations and by inspection recognize when the equations will produce one solution, no solution, or infinitely many solutions.

Checking for Understanding

Plant A and Plant B are on different watering schedules. This affects their rate of growth. Plant A started at a height of 4 inches and grows 2 inches per week. Plant B started at 2 inches and grows at 4 inches per week.

- Create a table that represents the height of each plant for each week.
- Plot the graph of each table on the same coordinate plane.
- Where do the graphs of each plant growth intersect?
- What does this intersection represent?
- Write an equation to represent each plant's growth per week.
- What is the relationship between the point of intersection and the equation for each plant's growth?

Find the solution to each of the following system of equations.

a) $b = \frac{2}{3}a + 1$
 $b = -\frac{1}{3}a + 7$

b) $y = \frac{3}{7}x - 4$
 $y = \frac{3}{7}x + 1$

Return to: [Standards](#)

Functions

Define, evaluate, and compare functions.

NC.8.F.1 Understand that a function is a rule that assigns to each input exactly one output.

- Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output.
- Recognize functions given a table of values or a set of ordered pairs.

Clarification

This standard starts to develop the definition of a function as a rule that produces a unique output for each input. This understanding has been building through students' work with ratio and proportional relationships. In 8th grade, functions will be represented as tables, graphs, and equations.

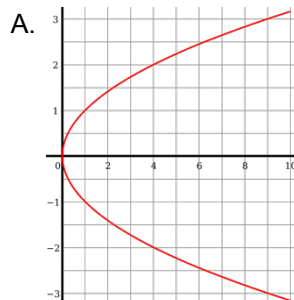
Function notation is not an expectation in 8th grade.

Recognizing functions from graphs, tables, or a set of ordered pairs.

Students identify graphs, tables or a set of ordered pairs that represent a function. These graphs, tables, and ordered pairs can represent nonlinear functions as well as linear functions. Students justify their reasoning using the definition of a function and characteristics from different representations of the function. If the students are exposed to the vertical line test, the students are expected to explain why it works.

Checking for Understanding

Which of the following are functions? Explain your reasoning for each.



B.

x	y
0	0
3	2
5	4
7	6
9	4

C. (-5,2), (3,-5), (-3,2), (0,6), (-3,-2), (-5,6)

Return to: [Standards](#)

Define, evaluate, and compare functions.

NC.8.F.2 Compare properties of two linear functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Clarification

This standard focuses on comparing linear functions that may be represented in different ways.

Keeping consistent with other function standards:

- Linear functions represented algebraically must be given in slope-intercept form.
- Functions represented with a verbal description must be able to be written directly into slope-intercept form.

Students may convert functions to different representations to assist in their comparison.

Students describe the comparison of linear functions qualitatively (with words) and quantitatively (with numbers) by discussing and analyzing:

- Intercepts
- Rates of change
- Points of intersection
- Other points important to the context

Checking for Understanding

Sandra is looking to buy decorative doughnuts for a party. She is considering two local bakers. She looks up two bakers online to find how much each one charges. Here are the advertisements she saw online.

Ms. Spellings Donut Shop

Try out this week's special!
All decorative donuts are on sale for \$1.62 each with an \$8 boxing fee!

Phil'd Up Doughnuts

Boxing fee included in price!

Decorative Doughnuts	Price
2	\$9
4	\$13
6	\$17
8	\$21
10	\$25
12	\$29

- What is the rate of change and the boxing fee for each doughnut shop?
- If the total cost was Sandra's biggest concern, who should she buy from? Justify your answer mathematically.

Return to: [Standards](#)

Define, evaluate, and compare functions

NC.8.F.3 Identify linear functions from tables, equations, and graphs.

Clarification

Students identify a linear function from a variety of representations, including tables, equations and graphs. As with all functions, students should view a linear function as describing a relationship between quantities. A linear function is unique from other functions because of the characteristics of its relationships, namely having a constant additive rate of change.

Identifying a linear function comes from considering the rate of change between points that are a part of the function. Students build an understanding of linear functions by:

- Knowing that the rate of change of the function is the ratio of change in the output to the change in the input.
- Finding the rate of change between all points and showing that these rates of change are equivalent.
- Finding the rate of change between equally spaced points and showing that these rates are the same and form an additive pattern for each quantity (constant additive rate of change).

For example: Compare the function $y = \frac{1}{2}x - 3$ with the function $y = x^2 + 1$ to determine if the functions represent a linear relationship.

Sample answer: Using a table to compare.

$y = \frac{1}{2}x - 3$		$y = x^2 + 1$	
x	y	x	y
0	-3	0	1
2	-2	3	10
4	-1	6	37
6	0	9	82

In the first function, $y = \frac{1}{2}x - 3$, the x's increase by 2, the y's increase by the constant pattern of +1. This gives a constant additive rate of change of 1 unit of y for each 2 units of x. This function is a linear function.

Note: As long as the x's change by a constant additive pattern, the y's will change at a constant additive pattern.

In the second function, $y = x^2 + 1$, as the x's increase by 3, the y's increase but not at a constant additive pattern. This function is nonlinear.

Students develop ways to see these patterns in the rate of change in both tables and graphs and understand how a constant additive rate of change would be represented in a linear equation. Students may see other patterns in the rates of change as other families of functions have different patterns.

In 8th grade, students identify all mathematical functions that are not linear as nonlinear functions.

Checking for Understanding

Janice and Kim noticed that both proportional relationships and linear functions form a straight line when graphed. Janice claims that all linear functions are also proportional relationships. Kim disagrees and tells Janice that she has it backwards, that all proportional relationships are linear functions.

- Who is correct? Explain.
- A counterexample is when a specific example is given that disproves a claim. Create a counterexample to disprove the false claim. Explain how your example disproves the claim.

Devon is looking at the following table. He is supposed to determine if the table represents a linear function. Devon believes that it is a linear function because the numbers in the y column form a pattern of decreasing by 5 from one row to the next.

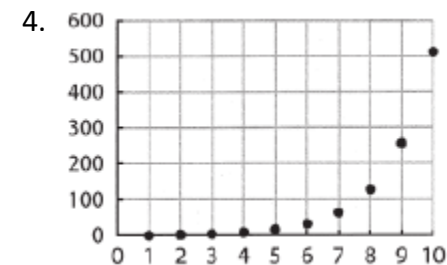
x	y
-3	0
-1	-5
0	-10
1	-15

However, this table does not represent a linear function.

- Explain to Devon why the table does not represent a linear function.
- Describe how you could change the values in the table so that it would represent a linear function.

Determine if the functions listed below are linear or non-linear. Explain your reasoning.

- $y = -2x^2 + 3$
- $y = 0.25 + 0.5(x - 2)$
- $y = x(3 - x) + 1$



Return to: [Standards](#)

Use functions to model relationships between quantities.

NC.8.F.4 Analyze functions that model linear relationships.

- Understand that a linear relationship can be generalized by $y = mx + b$.
- Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph.
- Construct a graph of a linear relationship given an equation in slope-intercept form.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y -intercept of its graph or a table of values.

Clarification

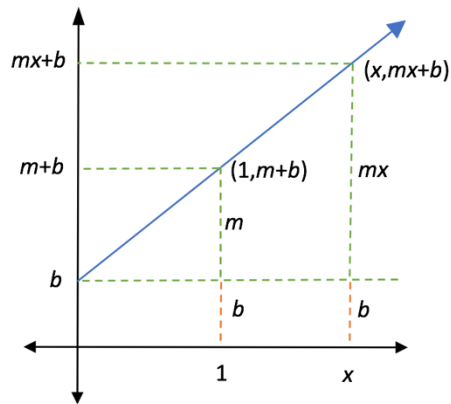
Analyzing linear functions is a major focus for 8th grade. In previous grades, students have worked with proportional relationships, which are a subset of linear functions. This means that some of the properties of a proportional relationship can be generalized to linear functions, while some properties are specific to proportional relationships.

Understand that a linear relationship can be generalized by $y = mx + b$.

In previous grades, students created tables, graphs, and equations from proportional relationships. Students can build from this previous knowledge to the larger generalization of linear functions.

In 7th grade, students learned that in a proportional relationship, the y -coordinate when $x = 1$ is the unit rate and built an understanding of the equation of a proportional relationship (see NC.7.RP.2).

x (input)	y (output)
0	b
1	$m + b$
2	$m \cdot 2 + b$
3	$m \cdot 3 + b$
...	...
x	$m \cdot x + b$



In 8th grade, students build on this understanding to develop the linear equation in slope-intercept form. The initial small triangle starts with sides of 1 and m . As the triangle is scaled by a factor of x , the sides of the new triangle become x and mx . However, in order to find the coordinate that is on the line, the y -coordinate must be increased by b , since the triangle is starting off the x -axis.

This gives the coordinate of $(x, mx + b)$. This means that when the input is x , the output is $mx + b$, producing the function, $y = mx + b$. The same generalization can be seen in the table.

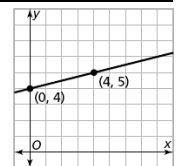
In 8th grade, the only required form of a linear equation is slope-intercept form. Students will not be asked to work with other forms of a linear equation or be asked to change the other forms to slope-intercept form.

Checking for Understanding

Write an equation that models the linear relationship in the table.

x	y
-2	8
0	2
1	-1

Write an equation that models the linear relationship in the graph.



Write an equation for the line that has a rate of change of $\frac{1}{2}$ and passes through the point $(-2, 5)$.

The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car's navigation system (GPS). Write a function for the cost in dollars, c , and the number of days, d , the car was rented.

Use functions to model relationships between quantities.

NC.8.F.4 Analyze functions that model linear relationships.

- Understand that a linear relationship can be generalized by $y = mx + b$.
- Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph.
- Construct a graph of a linear relationship given an equation in slope-intercept form.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y -intercept of its graph or a table of values.

Clarification

Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph.

Students find the rate of change between two points. These two points may be given in a table, on the coordinate plane, or as ordered pairs.

Students will also find the initial value in a linear function represented as an equation, table, or graph. Having found the rate of change and the initial value, students should be able to write a linear function that models the situation given.

This standard does not imply that students should memorize the slope formula or use a purely algebraic approach to find an equation. Students should be able to use patterns and other representations to find the initial value and rate of change.

Construct a graph of a linear relationship given an equation in slope-intercept form.

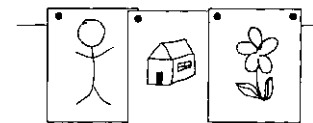
Students create the graphical representation of a linear function, given a linear equation in slope-intercept form, using the initial value and rate of change. Students will choose an appropriate scale to graph a linear function.

Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y -intercept of its graph or a table of values.

Students give meaning to the rate of change and the initial value of a linear function based on a context. The linear function can be given in a variety of representations, including verbal descriptions, tables, graphs, and equations. Students use terms such as slope and y -intercept to describe a graphical representation of a linear function and correlate their meaning to the rate of change and initial value, where the input is 0. Students should use the units from a context appropriately in their description of rate of change and initial value.

Checking for Understanding

Children's pictures are hung in a line as seen in the picture. Pictures that are hung next to each other share a tack.



- Describe the rate of change based on this context.
- How many tacks would be needed for 28 pictures?

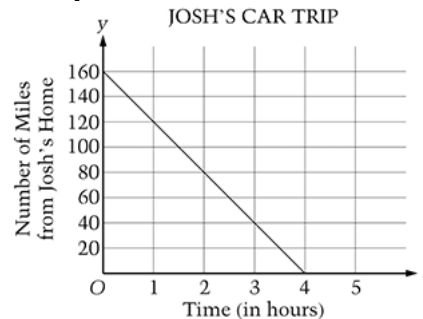
Adapted from NAEP – Released Item (1992) **Question ID:** 1992-8M7 #8 M045201

A leaf falls 18 feet from a branch to the ground at a rate of 5 feet every 2 seconds. Determine whether each statement about the leaf is true. If it is false, change the statement so that it is true.

Statement	T/F
The initial height of the leaf is 18 feet.	
The leaf falls at a rate of $\frac{2}{5}$ foot every 1 second.	
The leaf is 3 feet above the ground after 6 seconds.	

Adapted from SBAC Mathematics Practice Test Scoring Guide Grade 8 p. 11

The linear graph below describes Josh's car trip from his grandmother's home directly to his home.



- What is the distance from Josh's grandmother's home to his home?
- How long did it take for Josh to make the trip?
- What was Josh's average speed for the trip? Explain how you know.

Adapted from NAEP – Released Item (2011) **Question ID:** 2011-8M8 #15 M169901

Use functions to model relationships between quantities.

NC.8.F.5 Qualitatively analyze the functional relationship between two quantities.

- Analyze a graph determining where the function is increasing or decreasing; linear or non-linear.
- Sketch a graph that exhibits the qualitative features of a real-world function.

Clarification

Students focus on describing the characteristics of linear and nonlinear real-world functions.

Analyze a graph determining where the function is increasing or decreasing; linear or non-linear.

Students describe specific sections of a graph over which the output is increasing, decreasing, or remaining the same. Students describe which sections of the graph are linear and which are non-linear. Students use verbal descriptions in terms of the independent variable to define sections of a graph. For example, a student may say, “The function increases when x is between 3 and 7,” or “The average temperature rose between March and July.” Students are not expected to use compound inequalities in 8th grade when describing sections of a graph.

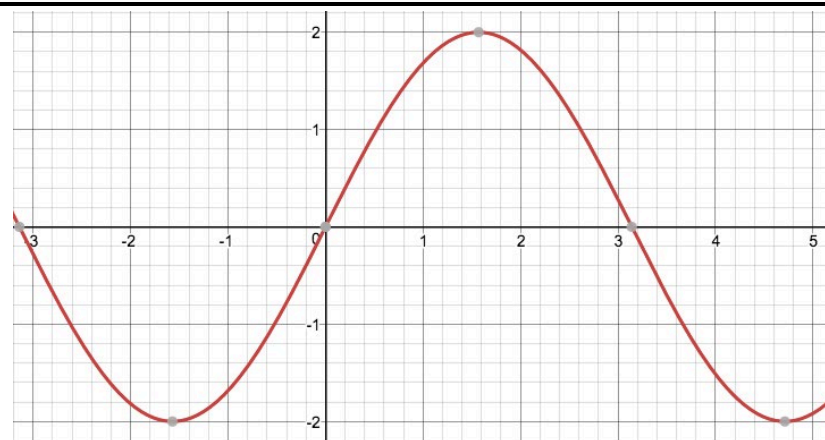
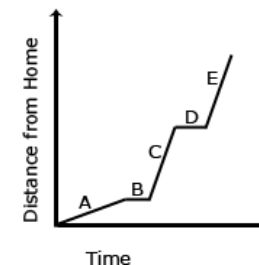
Students support their claims with mathematical reasoning.

Sketch a graph that exhibits the qualitative features of a real-world function.

Students sketch a graph based on the information and context provided. These graphs may be composed of sections of different linear relationships with corresponding rates of change. Information will be provided giving either the location of the boundary for each section or the duration in which a particular rate of change should be applied.

Checking for Understanding

The graph shows John’s trip to school. He walks to Sam’s house and, together, they ride a bus to school. The bus stops once before arriving at school. Describe how each part A – E of the graph relates to the story.



Looking at this graph, describe:

- Where the graph is increasing,
- Where the graph is decreasing,
- Where the graph is not increasing or decreasing,
- Any areas that appear to be linear.

Return to: [Standards](#)

Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

NC.8.G.2 Use transformations to define congruence:

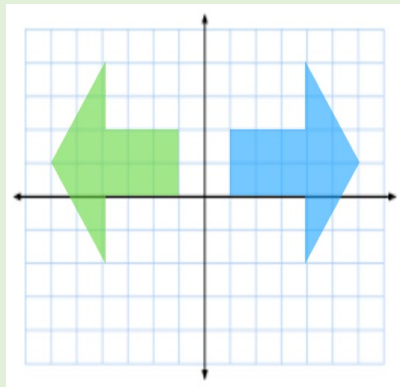
- Verify experimentally the properties of rotations, reflections, and translations that create congruent figures.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.
- Given two congruent figures, describe a sequence that exhibits the congruence between them.

Clarification

The focus on this standard is the conceptual development of the idea of **congruent figures**. Congruent figures have the same shape and size. Two figures in the plane are said to be congruent if there is a sequence of rigid motions that takes one figure onto the other. (Progressions for CCSSM Geometry, Grade 7-8, HS, 2016).

Given two congruent figures, students explore characteristics of the figures, such as lengths of line segments, angle measures, and parallel lines as they develop a definition for congruent figures. The coordinate plane can be used as a tool to develop understanding of this concept because it gives a visual image of the correspondence between the two figures.

In the following example, students should be able to compare the side lengths and angles created by adjacent sides for each figure and the correspondence of the sides and angles between the figures. They should also be able to determine what type of rigid transformation will map one figure onto the other. In this case, a reflection across the y -axis will map the green arrow onto the blue arrow and vice-versa.

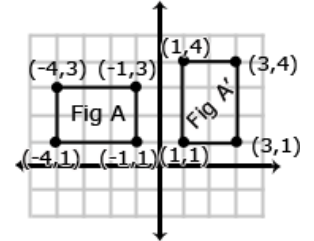


Students use mathematical language to distinguish the figures, noting that the figure prior to the transformation is called the **pre-image** (e.g. Figure A) and the post-transformation figure is called the **image** (e.g. Figure A').

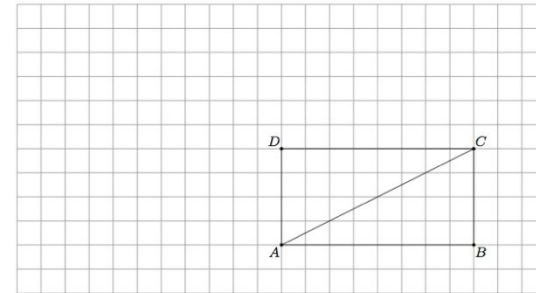
Students also examine two figures to identify the rigid transformation(s) that produced the image from the pre-image. Students recognize the symbol for congruency (\cong) and write statements of congruence.

Check for Understanding

Figure A and Figure A' are congruent. Identify at least three characteristics of the figures that supports this fact.



Below is a picture of rectangle ABCD with diagonal AC.



- Draw the image of triangle ACD when it is rotated 180° about vertex D . Call A' the image of point A under the rotation and C' the image of point C .
- Explain why $\overrightarrow{DA'} \cong \overrightarrow{DA}$ and why $\overrightarrow{DC'}$ is parallel to \overrightarrow{AB} .
- Show that $\triangle A'C'D'$ can be translated to $\triangle CAB$. Conclude that $\triangle ACD$ is congruent to $\triangle CAB$.
- Show that $\triangle ACD$ is congruent to $\triangle CAB$ with a sequence of translations, rotations, and/or reflections different from those chosen in parts (a) and (c).

Return to: [Standards](#)

Understand congruence and similarity using physical models, transparencies, or geometry software.

NC.8.G.3 Describe the effect of dilations about the origin, translations, rotations about the origin in 90 degree increments, and reflections across the x -axis and y -axis on two-dimensional figures using coordinates.

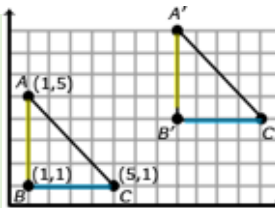
Clarification

The focus of this standard is on developing understanding of transformations using visualization, spatial reasoning, and geometric modeling. The coordinate plane is used as a tool to develop student understanding of transformations unifying the ideas of shape and location. Students study distance-preserving transformations (isometries) and dilations to aid in the development of the concepts of congruence and similarity, respectively. Students will also use point notation to describe the transformation of each point. For example, the point notation for a vertical translation of 3 units up would be described in point notation as $(x, y) \rightarrow (x, y + 3)$.

Isometries

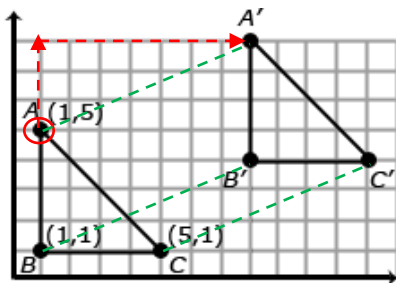
Isometries are also called rigid transformations because they preserve size and shape of a geometric figure. Translations, reflections, and rotations are **rigid** transformations.

For example, notice that the distance between the respective points in each figure is the same (i.e. there are 4 units between A and B, likewise between A' and B').



Translations

Translations move the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image.



For example, $\triangle ABC$ has been translated 7 units \rightarrow and 3 units \uparrow .
To get from A (1, 5) to A' (8, 8), move point A 7 units \rightarrow (from $x = 1$ to $x = 8$) and 3 units \uparrow (from $y = 5$ to $y = 8$).

Check for Understanding

Complete the table of transformation rules:

Type of Transformation	Point Notation
Vertical Translation (\uparrow)	$(x, y) \rightarrow (x, y + a)$
Vertical Translation (\downarrow)	$(x, y) \rightarrow (\quad , \quad)$
Horizontal Translation (\rightarrow)	$(x, y) \rightarrow (\quad , \quad)$
Horizontal Translation (\leftarrow)	$(x, y) \rightarrow (\quad , \quad)$
Reflection over x-axis	$(x, y) \rightarrow (\quad , \quad)$
Reflection over y-axis	$(x, y) \rightarrow (\quad , \quad)$
Rotation 90° (clockwise)	$(x, y) \rightarrow (\quad , \quad)$
Rotation 90° (counter-clockwise)	$(x, y) \rightarrow (\quad , \quad)$
Rotation 180° (clockwise)	$(x, y) \rightarrow (\quad , \quad)$
Rotation 180° (counter-clockwise)	$(x, y) \rightarrow (\quad , \quad)$
Rotation 270° (clockwise)	$(x, y) \rightarrow (\quad , \quad)$
Rotation 270° (counter-clockwise)	$(x, y) \rightarrow (\quad , \quad)$
Dilation (Scale up)	$(x, y) \rightarrow (\quad , \quad)$
Dilation (Scale down)	$(x, y) \rightarrow (\quad , \quad)$

Understand congruence and similarity using physical models, transparencies, or geometry software.

NC.8.G.3 Describe the effect of dilations about the origin, translations, rotations about the origin in 90 degree increments, and reflections across the x -axis and y -axis on two-dimensional figures using coordinates.

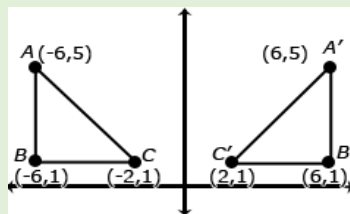
Clarification

Check for Understanding

Reflections

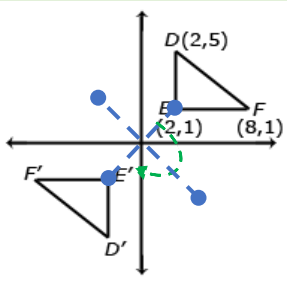
A reflection is the “flipping” of an object over a line, known as the “line of reflection”. In the 8th grade, students will only reflect over the x - and y -axis. Students recognize that when an object is reflected across the y -axis, the reflected x -coordinate is the opposite of the pre-image x -coordinate. Students can then infer what happens when reflected across the x -axis. Likewise, a reflection across the x -axis would change a pre-image coordinate $A(-6, 5)$ to the image coordinate of $A'(-6, -5)$. (**Point notation:** $(x, y) \rightarrow (x, -y)$)

The figure below is reflected across the y -axis. Compare point $A(-6, 5)$ to point $A'(6, 5)$, B to B' , and C to C' .



Rotations

A rotation is a transformation performed by “spinning” the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise up to 360° (at 8th grade, rotations will be around the origin and a multiple of 90°). Students know that lines passing through the point of rotation map onto themselves when rotated 180° , but lines that are rotated 180° that do not pass through the point of rotation create parallel lines.



For example, consider when triangle DEF is rotated 180° clockwise about the origin. The coordinates of triangle DEF are $D(2, 5)$, $E(2, 1)$, and $F(8, 1)$. When rotated 180° about the origin, the new coordinates are $D'(-2, -5)$, $E'(-2, -1)$ and $F'(-8, -1)$. In this case, each coordinate is the opposite of its pre-image (see figure). Notice that corresponding sides of the image and preimage are parallel.

Dilations

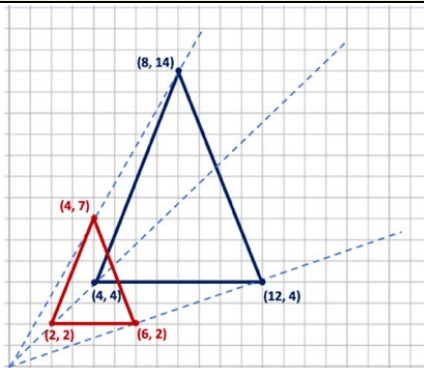
Understand congruence and similarity using physical models, transparencies, or geometry software.

NC.8.G.3 Describe the effect of dilations about the origin, translations, rotations about the origin in 90 degree increments, and reflections across the x -axis and y -axis on two-dimensional figures using coordinates.

Clarification

Building on understanding of scale factor in earlier grades, students learn that a dilation is a transformation that moves each point along a ray which starts from a fixed center and multiplies distances from this center by a common scale factor. Dilations enlarge (scale factors greater than one) or reduce (scale factors less than one) the size of a figure by the specified factor.

Students recognize the relationship between the coordinates of the pre-image, the image and the scale factor for a dilation from the origin. In 8th grade, dilations are limited from the origin.



Check for Understanding

Return to: [Standards](#)

Understand congruence and similarity using physical models, transparencies, or geometry software.

NC.8.G.4 Use transformations to define similarity.

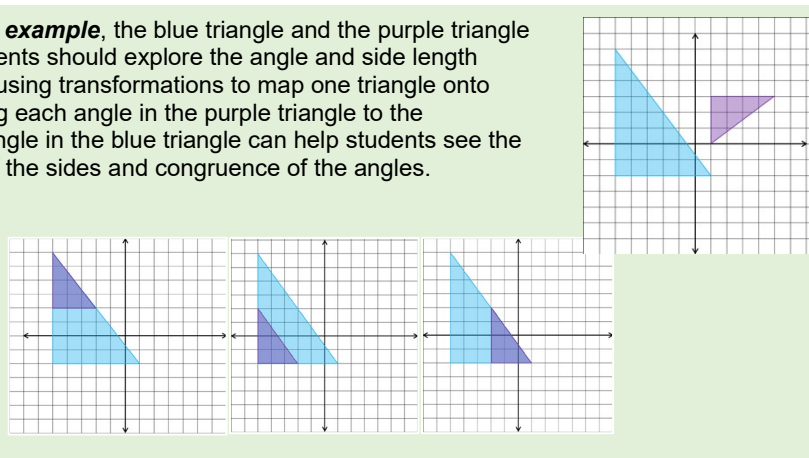
- Verify experimentally the properties of dilations that create similar figures.
- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.
- Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Clarification

The focus of this standard is on the conceptual development of **similar figures**. Similar figures have congruent corresponding angles and proportional corresponding side lengths. Two figures are *similar* if there is a sequence of dilations and rigid motions that places one figure directly on top of another (Progressions for CCSSM Geometry, Grade 7-8, HS, 2016).

Given two similar figures, students explore the proportional relationship between corresponding characteristics of the figures, such as lengths of line segments, and angle measures as they develop a definition for similar figures. In 8th grade, dilations are restricted from the origin. The coordinate plane can be used as a tool to develop understanding of this concept because it gives a visual image of the correspondence between the two figures. Additionally, transparencies or tracing paper can be used to show the congruency of the angles, while measurement tools can be used to examine the proportional relationships between edge lengths.

In the following example, the blue triangle and the purple triangle are similar. Students should explore the angle and side length relationships by using transformations to map one triangle onto another. Mapping each angle in the purple triangle to the corresponding angle in the blue triangle can help students see the proportionality of the sides and congruence of the angles.



Students use mathematical language to distinguish the figures, noting that the figure prior to the transformation is called the **pre-image** (e.g. Figure A) and the post-transformation figure is called the **image** (e.g. Figure A').

Students also examine two figures to identify the rigid transformation(s) that produced the image from the pre-image. Students recognize the symbol for similarity (\sim) and write statements of similarity.

Checking for Understanding

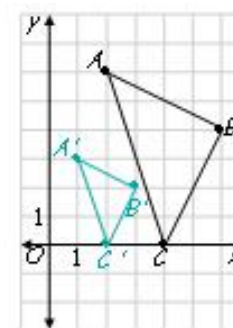
Triangle ABC undergoes a series of some of the following transformations to become triangle DEF:

- Rotation
- Reflection
- Translation
- Dilation

Is triangle DEF always, sometimes or never similar to triangle ABC? Justify your response. (Adapted from SBAC)

Given the dilation $\Delta ABC \rightarrow \Delta A'B'C'$, what is the scale factor?

- Verify proportionality using both the dimensions and the coordinates of the figures.
- Verify that the center of dilation is at the origin.



Return to: [Standards](#)

Analyze angle relationships.

NC.8.G.5 Use informal arguments to analyze angle relationships.

- Recognize relationships between interior and exterior angles of a triangle.
- Recognize the relationships between the angles created when parallel lines are cut by a transversal.
- Recognize the angle-angle criterion for similarity of triangles.
- Solve real-world and mathematical problems involving angles.

Clarification

This standard focuses on an **informal development** of the understanding of angle relationships in triangles and parallel lines. Students build on understandings of angle relationships in 7th grade and transformations in 8th grade.

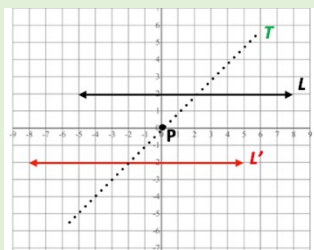
➤ **Interior and exterior angles in triangles**

- Students *informally discover* that the interior angles of a triangle have a sum of 180°.
- Students understand the relationships between interior and exterior angles of a triangle. Students know that:
 - every exterior angle is supplementary to its adjacent interior angle.
 - the measure of an exterior angle is equivalent to the sum of the remote interior angles.

➤ **Angles created when parallel lines are cut by a transversal**

- Building on understanding from transformations, students construct parallel lines and a transversal to examine the relationships between the created angles.
- Students recognize vertical angles, adjacent angles, and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles created by parallel lines and a transversal.

For example, since line L' is the image of line L after a 180° rotation about point P and line T , which passes through the point of rotation, maps onto itself after the same 180° rotation, then L and L' are parallel lines. This means that the angle relationships between lines T and L are the same angle relationships between lines T and L' .

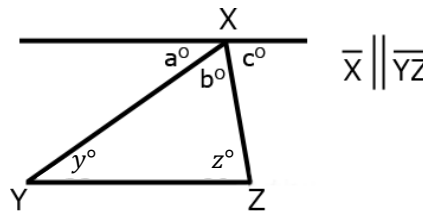


➤ **Angle-angle criterion for similarity of triangle.**

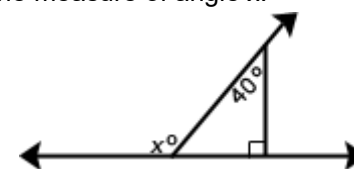
Students notice through exploration and conjecturing that there are an infinite number of triangles that can be created that have the same exact angle measurements and those triangles are therefore similar to each other and not necessarily congruent.

Check for Understanding

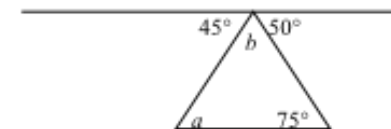
In the figure below line X is parallel to line YZ . How could you use this fact to show that the sum of the angles of $\triangle XYZ$ is 180°?



Write and solve an equation to find the measure of angle x .



Find the measures of angles a and b .



Josh drew $\triangle ABC$ with $m\angle A = 40^\circ$, $m\angle B = 60^\circ$ and $m\angle C = 80^\circ$. Howard drew $\triangle XYZ$ with $m\angle Y = 60^\circ$ and $m\angle Z = 80^\circ$. What can you say about the two triangles? Justify your response.

Return to: [Standards](#)

Understand and apply the Pythagorean Theorem.

NC.8.G.6 Explain the Pythagorean Theorem and its converse.

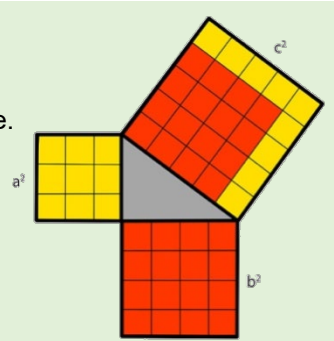
Clarification

The focus of this standard is on examining different models of the Pythagorean Theorem and its converse and showing understanding of how the models support the theorem. Students are NOT expected to prove the Pythagorean Theorem. However, students should be able to explain a proof provided that is within the scope of middle school mathematics.

Pythagorean Theorem: The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c).

Students can explain the Pythagorean Theorem using models. Students understand the connection between the Pythagorean Theorem and area.

For example, students understand that the area of the squares that form the legs is equivalent to the area of the square created by the hypotenuse. Students can explain verbally or rearrange the area of tiles of the smaller square to create the larger square. There are a variety of ways that students can explain understanding.

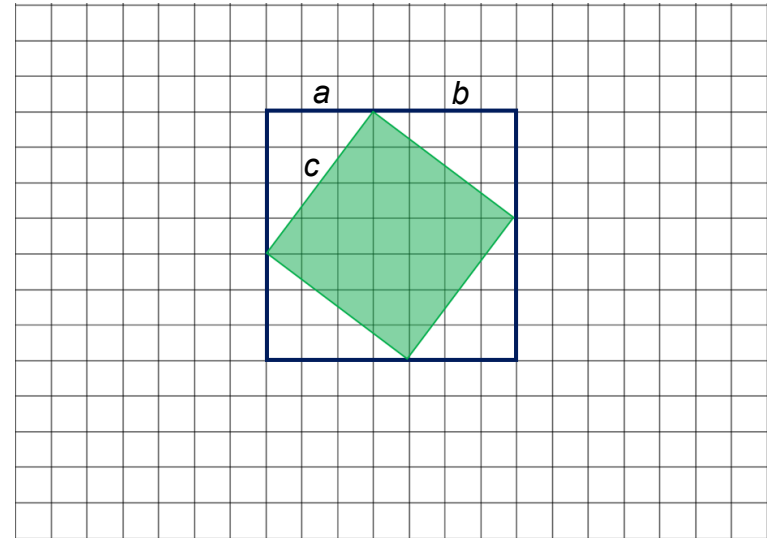


The Converse of the Pythagorean Theorem: If a triangle has sides of length a , b , and c and if $a^2 + b^2 = c^2$ then the angle opposite the side of length c is a right angle.

Students are able to determine whether a triangle is a right triangle by examining the relationship between the sides of a triangle. This standard should build from work with triangles in the 7th grade, where students determined if a triangle exists based on the relationship between the sides (NC.7.G.2).

Checking for Understanding

How does the following diagram support the Pythagorean Theorem? Explain.



Return to: [Standards](#)

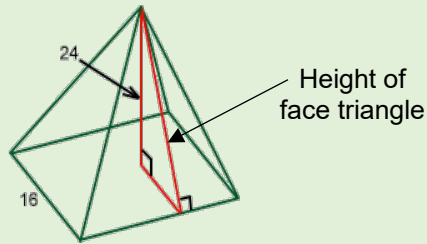
Understand and apply the Pythagorean Theorem.

NC.8.G.7 Apply the Pythagorean Theorem and its converse to solve real-world and mathematical problems.

Clarification

This standard focuses on the application of the Pythagorean Theorem. Students will apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in both two and three-dimensional objects.

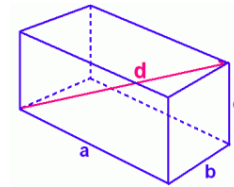
For example, The Pythagorean Theorem can be used to find the height of the triangles on the faces of the square pyramid, so that the surface area of the pyramid can be calculated.



Checking for Understanding

The Irrational Club wants to build a tree house. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the tree house be off the ground?

Find the length of d in the figure to the right if $a = 8$ in., $b = 3$ in. and $c = 4$ in.



The distance from Jonestown to Maryville is 180 miles, the distance from Maryville to Elm City is 300 miles, and the distance from Elm City to Jonestown is 240 miles. Do the three towns form a right triangle? Why or why not?

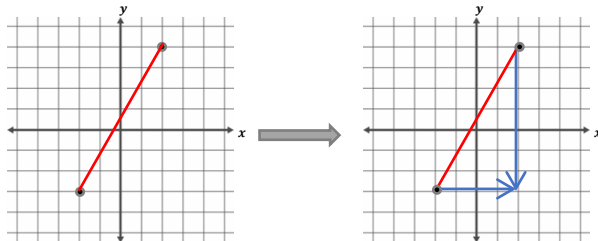
Return to: [Standards](#)

Understand and apply the Pythagorean Theorem.

NC.8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Clarification

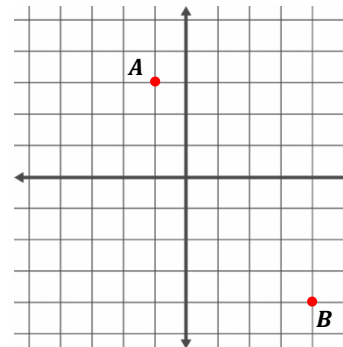
This standard uses the Pythagorean Theorem as an application to find the distance between two non-vertical and non-horizontal points on a line. Students build on work from 6th grade where they found distances between vertical and horizontal lines in the coordinate plane. Given two points, students are able to draw a line connecting the points and create a right triangle using the points. Students understand that the line segment between the two points is the length of the hypotenuse of the right triangle that can be formed. They also understand that the third vertex of the triangle is the intersection of the vertical and horizontal lines.



Note: The distance formula is NOT an expectation.

Checking for Understanding

Draw the right triangle where \overline{AB} is the hypotenuse? What is the length of \overline{AB} ?



Return to: [Standards](#)

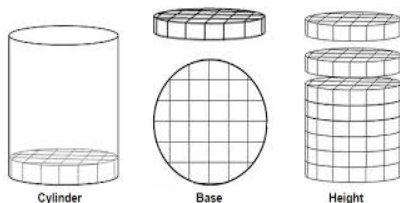
Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

NC.8.G.9 Understand how the formulas for the volumes of cones, cylinders, and spheres are related and use the relationship to solve real-world and mathematical problems.

Clarification

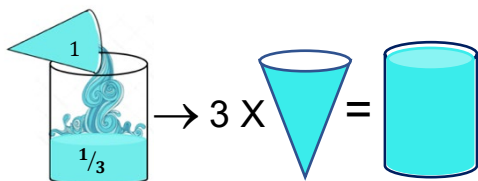
This standard focuses on the volume formulas for 3-dimensional shapes related to circles (cones, cylinders, and spheres). Students have already worked with volume of cubes and rectangular prisms. They apply these same understandings beginning with cylinders.

For a cylinder, students understand that the volume represents a “stack” or layers of area of the base. So, they apply the generalized formula $\rightarrow V = Bh$, where B represents the area of the base and h represents the height of the stack.



Base is a circle, so $A = \pi r^2$ $V = \pi r^2 h$

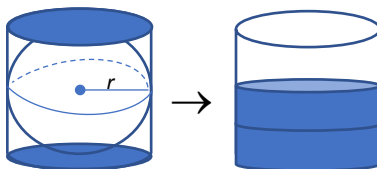
<https://www.texasgateway.org/resource/determining-volume-cones-and-cylinders>



For a cone, students understand that the volume of a cylinder is 3 times the volume of a cone having the same base area and height OR that the volume of a cone is $\frac{1}{3}$ the volume of a cylinder having the same base area and height.

Therefore, $V_{\text{cone}} = \frac{1}{3} \pi r^2 h$ OR $V_{\text{cone}} = \frac{\pi r^2 h}{3}$.

A sphere can be enclosed within a cylinder, which has the same radius and height of the sphere. If the sphere is flattened, it will fill $\frac{2}{3}$ of the cylinder.



Note: The height of the cylinder is twice the radius of the sphere.

Based on this model, students understand that the volume of a sphere is $\frac{2}{3}$ the volume of a cylinder with the same radius and height. The height of the cylinder is the same as the diameter of the sphere or $2r$.

We can derive the volume formula for a sphere:

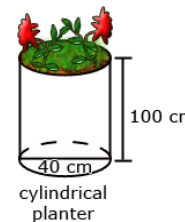
Beginning with the *cylinder* formula $\rightarrow V = \pi r^2 h$;

Multiply the *cylinder* formula by $\frac{2}{3} \rightarrow V = \frac{2}{3} \pi r^2 h$;

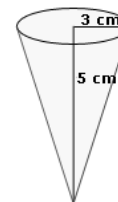
Substitute $2r$ for the height $\rightarrow V = \frac{2}{3} \pi r^2 (2r)$; Therefore, $V_{\text{sphere}} = \frac{4}{3} \pi r^3$.

Checking for Understanding

James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter’s volume.



How much yogurt is needed to fill the cone to the right? Express your answers in terms of π .



Approximately, how much air would be needed to fill a soccer ball with a radius of 14 cm?

Return to: [Standards](#)

Statistics and Probability

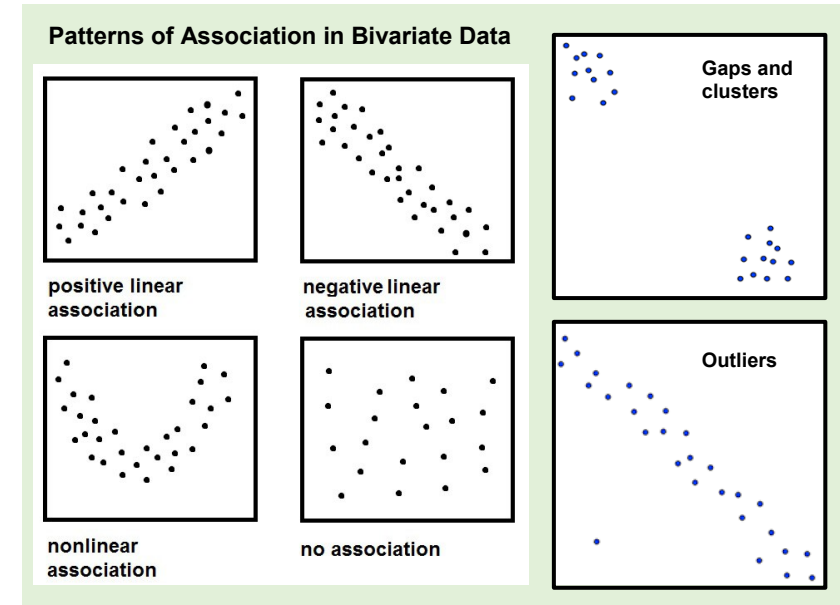
Investigate patterns of association in bivariate data.

NC.8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Clarification

This standard focuses on graphing and interpreting two-variable (bivariate) qualitative/measurement data. Students will build on previous experiences, such as graphing ordered pairs on the coordinate plane and analyzing the relationships between quantitative variables.

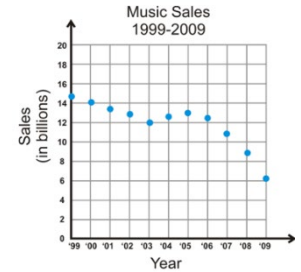
Students will extend their understanding by investigating patterns of association between the variables. This includes recognizing associations that are both linear and non-linear, which differ from no association. Students also recognize when clusters and/or gaps are present in the data. Finally, students can identify points that are deviations from associated data noting them as outliers.



Students can construct graphs by hand, using calculators, or through the use of computer software programs (i.e. Excel, GeoGebra, CODAP, etc.). Online tools, such as those at the [National Center for Educational Statistics](http://nces.ed.gov/ipeds/datacenter/natass/), can also be used to create a graph or generate data sets.

Checking for Understanding

The music sales (in billions) for the years 1999-2009 is displayed in the scatter plot. What are the pattern(s) of association for the music sales in the decade between 1999 and 2009? Explain.

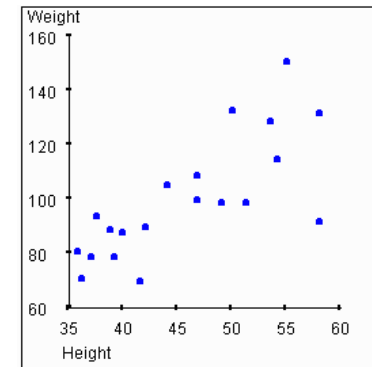


Source: CNN

Data for 10 students' Math and Science scores are provided in the chart. Describe what the data says about the association between the Math and Science scores.

Student #	1	2	3	4	5	6	7	8	9	10
Math Score	64	50	85	34	56	24	72	63	42	93
Science Score	68	70	83	33	60	27	74	63	40	96

The scatter plot represents the height and weight of a class of a sample of individuals from a local doctor's office. Describe the association any associations in the data, noting whether there are clusters, gaps or outliers.



Return to: [Standards](#)

Investigate patterns of association in bivariate data.

NC.8.SP.2 Model the relationship between bivariate quantitative data to:

- Informally fit a straight line for a scatter plot that suggests a linear association.
- Informally assess the model fit by judging the closeness of the data points to the line.

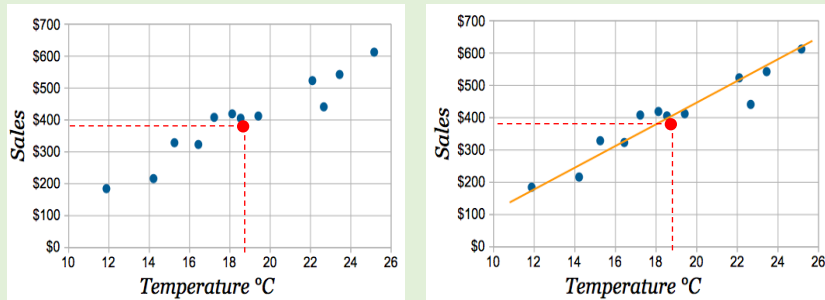
Clarification

This standard is a modeling standard. Students model linear relationships using graphs. Building on their understanding of statistical variation of univariate data, students extend this understanding to bivariate data.

Students understand that a straight line can represent a scatter plot with what appears to be a linear association. If a linear relationship is suspected, students draw the linear model that represents the *direction* of the association in the scatter plot minimizing the distance between the actual *y*-value and the predicted *y*-value (represented on the line) for each point.

The centroid (mean of *x*-values, mean of *y*-values) is a point on the estimated model line. Using this point as a pivot point can help students to determine the placement of the line. This is an informal understanding to assist students in the construction of the line. The line may or may not go through any or all of the data points. The use of linear regression is NOT expected.

For example, the following graph represents the ice cream sales of a local ice cream shop versus the noon temperature for 12 days. The association appears to be positive meaning that as the temperature increased, the ice cream sales increased also. The red dot (Figure 1) represents the average temperature and sales (18.7, 402). The line is drawn to reflect the direction (positive) of the association and as close as possible to all data points in the scatterplot.



<https://www.mathsisfun.com/data/scatter-xy-plots.html>

Furthermore, students notice any data values that fall outside the general pattern of associated data.

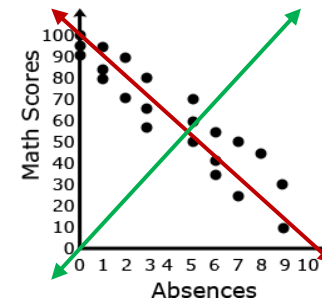
Checking for Understanding

Given data from students' math scores and absences:

- Make a scatterplot.
- Draw a linear model paying attention to the closeness of the data points on either side of the line.

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80

Given the scatter plot from students' math scores and absences, which line best models the association of the data? Explain.



Return to: [Standards](#)

Investigate patterns of association in bivariate data.

NC.8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate quantitative data, interpreting the slope and y-intercept.

Clarification

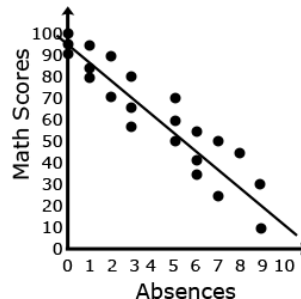
This standard is a modeling standard. Students model linear relationships using linear equations. Students can interpret the coefficient (slope) and constant (y-intercept) of the equation in the context of the problem.

This standard extends understandings from previous grade levels where students have graphed and created equations of quantitative relationships (NC.6.EE.9, NC.7.RP.2c).

Students have also interpreted the meaning of points (x, y) and quantities (rates) within proportional relationships (NC.7.RP.2d). Additionally, students apply this same understanding to non-proportional relationships (NC.8.F.4) where all points are collinear. This standard is extended to work with scatter plots, where the points generally are non-collinear, and students have to select appropriate points to write the equation of the line.

Checking for Understanding

Given the scatter plot and line for student math scores and absences, select two points to write the linear equation for the line. Interpret the slope and y-intercept in context of the problem.



Return to: [Standards](#)

Investigate patterns of association in bivariate data.

NC.8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

- Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
- Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Clarification

This standard extends understandings of bivariate numerical data to bivariate *categorical* data. Students understand that a two-way table provides a way to organize data between two categorical variables.

Students have experience with categorical data from elementary school and relative frequencies in 7th grade. They will extend this understanding to developing two-way tables of frequencies and relative frequencies.

Students will examine patterns of association in categorical data by examining the relative frequencies in a two-way table. Students recognize that similar proportions indicate that there is no association indicating similarity between populations.

Checking for Understanding

Kayla asked 10 students in her class whether they owned a dog, a cat or both. Complete the table below with the frequencies using the following relative frequencies:

- 40% of the students own a dog
- 30% of the students own a cat
- 10% of the students own both

		Dog		Totals
		Yes	No	
Cat	Yes			
	No			
Totals				10

Adapted from SBAC Released Items (Item #767, Grade 8)

All the students at a middle school were asked to identify their favorite academic subject and whether they were in 7th grade or 8th grade. Here are the results:

Favorite Subject by Grade

Grade	English	History	Math/Science	Other	Total
7 th Grade	38	36	28	14	116
8 th Grade	47	45	72	18	182
Totals	85	81	100	32	298

Is there an association between favorite academic subject and grade for students at this school? Support your answer by calculating appropriate row relative frequencies using the given data.

Taken from: *Illustrative Mathematics: What's Your Favorite Subject?*

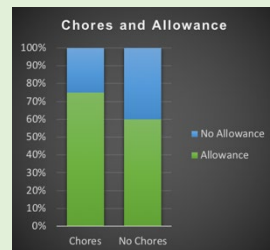
For example, the tables below represent data from a survey of 25 students to determine if there is an association between allowance and chores. Since there is a difference in the relative frequencies, there appears to be an association between doing chores and receiving allowance.

Frequency Table

	Allowance	No Allowance	Total
Chores	15	5	20
No Chores	3	2	5
Total	18	7	25

Relative Frequency Table

	Allowance	No Allowance
Chores	$\frac{15}{20} = .75$	$\frac{5}{20} = .25$
No Chores	$\frac{3}{5} = .60$	$\frac{2}{5} = .40$



The segmented bar graph is a visual display of the data. Since the green bar decreases for those not doing chores, there appears to be an association between doing chores and receiving allowance for this group of students. Additionally, not doing chores is associated with not receiving allowance.

Students DO NOT need to create segmented bar graphs; this visual is used to aid in understanding.

Return to: [Standards](#)