

ELEMENTS OF CHEMICAL REACTION ENGINEERING

FIFTH EDITION

H. SCOTT FOGLER



INTERNATIONAL SERIES IN THE
PHYSICAL AND CHEMICAL ENGINEERING SCIENCES

*Elements
of Chemical
Reaction
Engineering*

Fifth Edition

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Fifth Edition

H. SCOTT FOGLER

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and the Arthur F. Thurnau Professor
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Library of Congress Cataloging-in-Publication Data

Fogler, H. Scott, author.

Elements of chemical reaction engineering / H. Scott Fogler.—Fifth edition.

pages cm

Includes index.

ISBN 978-0-13-388751-8 (hardcover : alk. paper)

1. Chemical reactors. I. Title.

TP157.F65 2016

660'.2832—dc23

2015032892

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ISBN-13: 978-0-13-388751-8

ISBN-10: 0-13-388751-0

Dedicated to

Janet Meadors Fogler

*For her companionship, encouragement,
sense of humor, love, and support throughout the years*

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Preface

The man who has ceased to learn ought not to be allowed to wander around loose in these dangerous days.

M. M. Coady

A. Who Is the Intended Audience?

This book and interactive Web site is intended for use as both an undergraduate-level and a graduate-level text in chemical reaction engineering. The level will depend on the choice of chapters, the *Professional Reference Shelf (PRS)* material (from the companion Web site) to be covered, and the type and degree of difficulty of problems assigned. It was written with today's students in mind. It provides instantaneous access to information; does not waste time on extraneous details; cuts right to the point; uses more bullets to make information easier to access; and includes new, novel problems on chemical reaction engineering (e.g., solar energy). It gives more emphasis to chemical reactor safety (Chapters 12 and 13) and alternative energy sources—solar (Chapters 3, 8, and 10) and biofuel production (Chapter 9). The graduate material on topics such as effectiveness factors, non-ideal reactors, and residence time distribution is in Chapters 14–18 and now includes the software solutions for maximum mixedness and segregation models. A draft version of this book was class tested at the University of Michigan and other leading universities; then the text was further revised, taking into account the suggestions of more than 100 students. Much of the material was revised and reworked based on that feedback.

B. What Are the Goals of This Book?

B.1 To Have Fun Learning Chemical Reaction Engineering (CRE)

Chemical reaction engineering (CRE) is one of two core courses that is unique to chemical engineering and that separates the chemical engineer from other

engineers. CRE is a great subject that is fun to learn and is the heart of chemical engineering. I have tried to provide a little Michigan humor as we go. Take a look at the humorous YouTube videos (e.g., “Black Widow” or “Chemical Engineering Gone Wrong”) that illustrate certain principles in the text. These videos were made by chemical engineering students at the universities of Alabama and Michigan. In addition, I have found that students very much enjoy the Interactive Computer Games (ICGs) that, along with the videos, are linked from the CRE homepage (<http://www.umich.edu/~elements/5e>).

B.2 To Develop a Fundamental Understanding of Reaction Engineering

The second goal of this book is to help the reader clearly understand the fundamentals of CRE. This goal is achieved by presenting a structure that allows the reader to solve reaction engineering problems through reasoning rather than through memorization and recall of numerous equations and the restrictions and conditions under which each equation applies. The algorithms presented in the text for reactor design provide this framework, and the homework problems give the reader practice using the algorithms described in Figures P-1 and P-2, shown in Section C. The conventional homework problems at the end of each chapter are designed to reinforce the principles in the chapter. These problems are about equally divided between those that can be solved with a calculator and those that require a personal computer with a numerical software package, such as Polymath, AspenTech, or COMSOL.



To give a reference point as to the level of understanding of CRE required in the profession, a number of reaction engineering problems from the California Board of Registration for Civil and Professional Engineers—Chemical Engineering Examinations (PECEE) are included in the text.¹ Typically, these problems should each require approximately 30 minutes to solve.

Finally, the companion Web site should greatly facilitate learning the fundamentals of CRE because it includes Summary Notes of the material in each chapter, PowerPoint slides of class lecture notes, additional examples, expanded derivations, and self-tests. A complete description of these learning resources is in Appendix I.

B.3 To Enhance Thinking Skills

A third goal of this text is to enhance *critical thinking skills* and *creative thinking skills*. How does the book help enhance your critical and creative thinking skills? We discuss ways to achieve this enhancement in Section I of the Preface.

¹ The permission for use of these problems—which, incidentally, may be obtained from the Documents Section, California Board of Registration for Civil and Professional Engineers—Chemical Engineering, 1004 6th Street, Sacramento, CA 95814, is gratefully acknowledged. (Note: These problems have been copyrighted by the California Board of Registration and may not be reproduced without its permission.)

C. What Is the Structure of CRE?

C.1 What Are the Concepts that Form the Foundation of CRE?

The strategy behind the presentation of material is to build continually on a few basic ideas in CRE to solve a wide variety of problems. These ideas, referred to as the Pillars of Chemical Reaction Engineering (Figure P-1), are the foundation on which different applications rest. They represent not only components of chemical reaction analysis, but also the physical phenomena of diffusion and contacting that affect chemical reactor design.

Figure P-2 shows the first building blocks of CRE and the primary algorithm that allows us to solve CRE problems through logic rather than memorization. We start with the *Mole Balance Building Block* (Chapter 1) and then place the other blocks one at a time on top of the others until we reach the *Evaluate Block* (Chapter 5), by which time we can solve a multitude of isothermal CRE problems. As we study each block we need to make sure we understand everything in that block and don't leave anything out so we don't wind up with a cylindrical block. A tower containing cylindrical blocks would be unstable and would fall apart as we study later chapters. See the end of Chapter 1 lecture notes on the CRE Web site to see the tower of CRE fall if you have blocks with rounded edges.

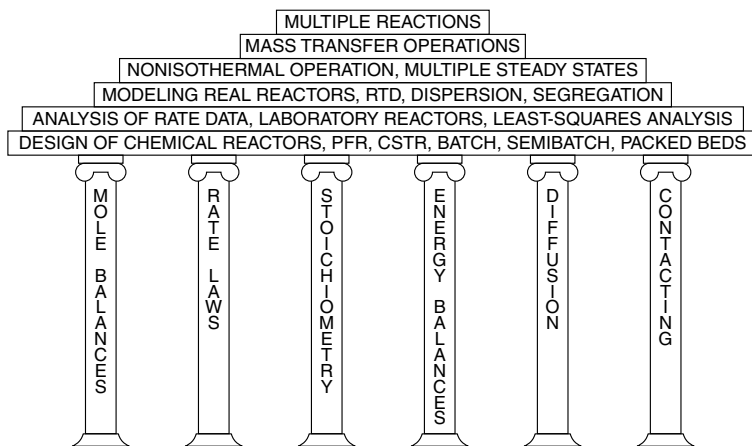


Figure P-1 Pillars of Chemical Reaction Engineering.

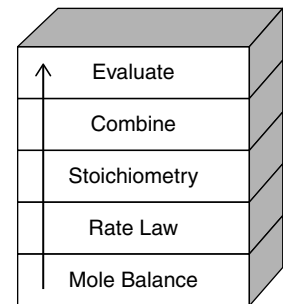


Figure P-2 Building blocks.

From these pillars and building blocks, we construct our CRE algorithm:

Mole Balance + Rate Laws + Stoichiometry + Energy Balance + Combine → Solution

With a few restrictions, the contents of this book can be studied in virtually any order after students have mastered the first six chapters. A flow diagram showing the possible paths is shown in Figure P-3.

The reader will observe that although metric units are used primarily in this text (e.g., kmol/m^3 , J/mol), English units are also employed (e.g., lb_m/ft^3 , Btu). This choice is intentional! We believe that whereas most papers published today use the metric system, a significant amount of reaction engineering data exists in the older literature in English units. Because engineers will be faced with extracting information and reaction rate data from older literature as well as from the current literature, they should be equally at ease with both English and metric units.

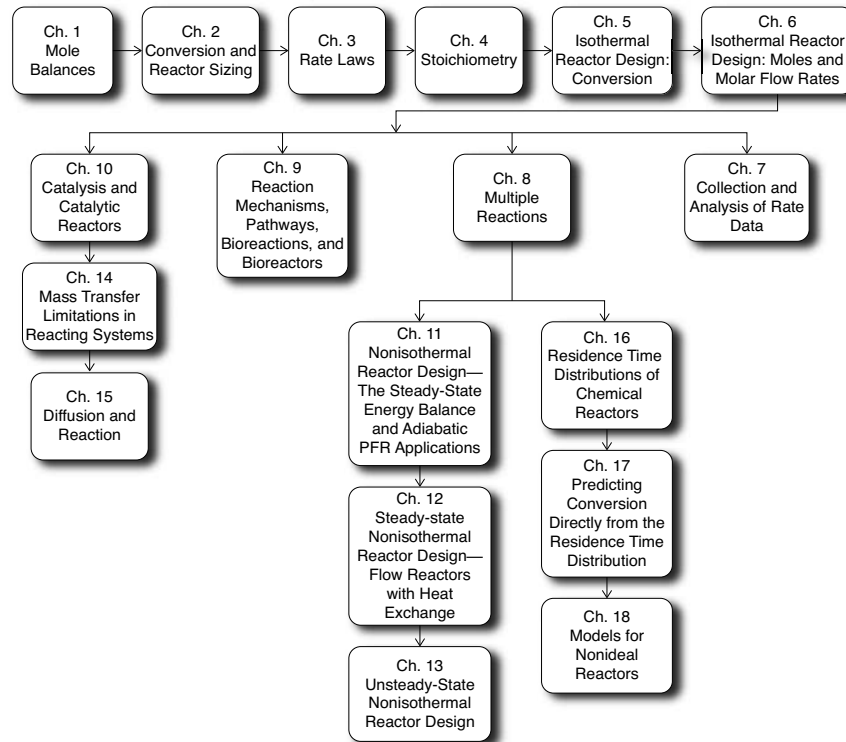


Figure P-3 Sequences for study using this text.

C.2 What Is the Sequence of Topics in which This Book Can Be Used?

Table P-1 shows examples of topics that can be converged in a graduate course and an undergraduate course. In a four-hour undergraduate course at the University of Michigan, approximately thirteen chapters are covered in the following order: Chapters 1 through 7 (Exam 1); Chapters 8, 11, and 12 (Exam 2); and Chapter 13 and parts of Chapters 9 and 10 (Exam 3).

There are notes in the margins, which are meant to serve two purposes. First, they act as guides or commentary as one reads through the material. Second, they identify key equations and relationships that are used to solve CRE problems.

D. What Are the Components of the CRE Web Site?

The companion, interactive Web site material is an updated version of the CRE Web site and is a novel and unique part of this book. The main purposes of the Web site are to serve as an enrichment resource and as a “professional reference shelf.” The home page for the CRE Web site (<http://www.umich.edu/~elements/5e/index.html>) is shown in Figure P-4. For discussion of how to use the Web site and text interactively, see Appendix I.

TABLE P-1 UNDERGRADUATE/GRADUATE COVERAGE OF CRE

<i>Undergraduate Material/Course</i>	<i>Graduate Material/Course</i>
Mole Balances (Ch. 1)	Short Review (Ch. 1–8, 11–12)
Smog in Los Angeles Basin (PRS Ch. 1)	Collision Theory (PRS Ch. 3)
Reactor Staging (Ch. 2)	Transition State Theory (PRS Ch. 3)
Hippopotamus Stomach (PRS Ch. 2)	Molecular Dynamics (PRS Ch. 3)
Rate Laws (Ch. 3)	Aerosol Reactors (PRS Ch. 4)
Stoichiometry (Ch. 4)	Multiple Reactions (Ch. 8):
Reactors (Ch. 5):	Fed Membrane Reactors
Batch, PFR, CSTR, PBR	Bioreactions and Reactors (Ch. 9, PRS 9.3–9.5)
Reactors (Ch. 6):	Polymerization (PRS Ch. 9)
Semibatch, Membrane	Co- and Countercurrent Heat Exchange
Data Analysis: Regression (Ch. 7)	(Ch. 12)
Multiple Reactions (Ch. 8)	Radial and Axial Gradients in a PFR
Bioreaction Engineering (Ch. 9)	COMSOL (Ch. 12)
Adiabatic Reactor (Ch. 11)	Reactor Stability and Safety (Ch. 12, 13, PRS
Steady-State Heat Effects (Ch. 12):	12.3)
PFR and CSTR with and without a	Runaway Reactions (PRS, Ch. 12)
Heat Exchanger	Catalyst Deactivation (Ch. 10)
Multiple Steady States	Residence Time Distribution (Ch. 16, 17)
Unsteady-State Heat Effects (Ch. 13)	Models of Real Reactors (Ch. 18)
Reactor Safety	Applications (PRS): Multiphase Reactors,
Catalysis (Ch. 10)	CVD Reactors, Bioreactors

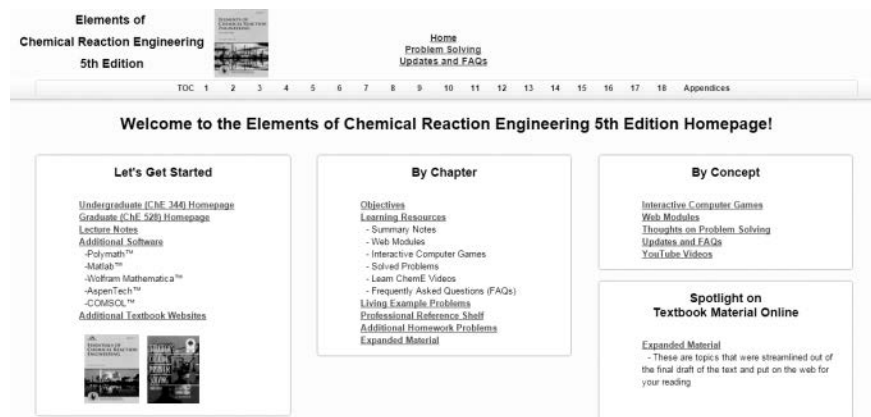


Figure P-4 Screen shot of the book's companion Web site (<http://www.umich.edu/~elements/5e/index.html>).

The objectives of the Web site are fourfold:

- (1) To facilitate the learning of CRE by using the companion Web site to actively address the Felder/Solomon Inventory of Learning Styles 7 discussed in Web Appendix I
- (2) To provide additional technical material

- (3) To provide tutorial information and self-assessment exercises
- (4) To make the learning of CRE fun through the use of interactive games

The following sections (D.1 through D.4) are listed at the end of most chapters and can be accessed from each chapter in the companion Web site.²

D.1 Expanded Material

The expanded material consists of material that was removed from the printed text and moved to the Web site to reduce the size and weight of the physical textbook. Reducing the weight makes it easier for the students to carry the book with them at all times, such as while on the campus shuttle or while eating at the cafeteria or the student union. The expanded material includes derivations, examples, and novel applications of CRE principles.

D.2 Learning Resources

The Learning Resources give an overview of the material in each chapter and provide extra explanations, examples, and applications to reinforce the basic concepts of CRE; the Learning Resources are described in Appendix I. The CRE Web site includes the following additional resources:

1. *Summary Notes and PowerPoint Slides*

The Summary Notes give an overview of each chapter and provide on-demand additional examples, derivations, and audio comments, as well as self-tests to assess each reader's understanding of the material. Copies of the PowerPoint slides are available from this course taught at the University of Michigan as well as those from Professor Mary Kraft's class at the University of Illinois.

2. *What Entertainment Is on the Web Site?*

2.A. *YouTube Videos*

We have included links to humorous YouTube Videos made by students in Professor Alan Lane's 2008 chemical reaction engineering class at the University of Alabama, as well as videos from the University of Michigan 2011 class. Specifically, check out "The Black Widow Murder Mystery" (Chapter 3), "CRF Reactor," and "Diet Coke and Mentos" (Chapter 4); learn a new dance and song ("CSTR" to the tune of "YMCA"); hear a rap song ("Find Your Rhythm," an "Ice Ice Baby" remix (Chapter 5)); and watch University of Michigan videos, including the ever-popular chemical engineering classic, "Reaction Engineering Gone Wrong."

2.B. *Interactive Computer Games (ICGs)*

Students have found the Interactive Computer Games to be both fun and extremely useful to review the important chapter concepts and then apply them to real problems in a unique and

² <http://www.ncsu.edu/felder-public/ILSdir/styles.htm>

entertaining fashion. The following ICGs are available on the Web site:

- Quiz Show I (Ch. 1)
- Reactor Staging (Ch. 2)
- Quiz Show II (Ch. 4)
- Murder Mystery (Ch. 5)
- Tic Tac (Ch. 5)
- Ecology (Ch. 7)
- The Great Race (Ch. 8)
- Enzyme Man (Ch. 9)
- Catalysis (Ch. 10)
- Heat Effects I (Ch. 12)
- Heat Effects II (Ch. 12)

As you play these interactive games, you will be asked a number of questions related to the corresponding material in the textbook. The ICG keeps track of all the correct answers and at the end of the game displays a coded performance number that reflects how well you mastered the material in the text. Instructors have a manual to decode the performance number.

2.C. *Web Modules*

The Web Modules are a number of examples that apply key CRE concepts to both standard and nonstandard reaction engineering problems (e.g., glow sticks, the use of wetlands to degrade toxic chemicals, and pharmacokinetics of death from a cobra bite). The Web Modules can be loaded directly from the CRE Web site (http://www.umich.edu/~elements/5e/web_mod/index.html).

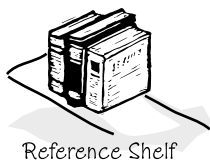
3. *Solved Problems*

A number of solved problems are presented along with problem-solving heuristics. Problem-solving strategies and additional worked example problems are available in the Problem Solving section of the CRE Web site.

D.3 Professional Reference Shelf

This section of the CRE Web site contains

1. Material from the fifth edition of *Elements of Chemical Reaction Engineering* that is not included in the printed textbook.
2. Material that is important to the practicing engineer, such as details of the industrial reactor design for the oxidation of SO_2 and design of spherical reactors and other material that is typically not included in the majority of chemical reaction engineering courses but is included here.



E. Why Do We Assign Homework Problems?

The working of homework problems facilitates a true understanding of CRE. After reading a chapter the student may feel they have an understanding of the material. However, when attempting a new or slightly different application of CRE in a homework problem, students sometimes need to go back and re-read

different parts of the chapter to get the level of understanding needed to eventually solve the homework problem.

The end-of-chapter problems numbered “2” (e.g., P3-2_A, P11-2_B) ask questions about the example problems in that chapter. These example problems are a key resource. These number-2-level problems should be worked before tackling the more challenging homework problems in a given chapter. The subscript letter (A, B, C, or D) after each problem number denotes the difficulty of the problem (i.e., A = easy; D = difficult).

F. What Is a Living Example Problem (LEP)?

The example problems that use an Ordinary Differential Equation (ODE) solver (e.g., Polymath) are referred to as “Living Example Problems” or “LEPs” because students can load the Polymath program directly onto their own computers in order to study the problem. Students are encouraged to change parameter values and to “play with” the key variables and assumptions. Using the LEPs to explore the problem and asking “what if...?” questions provide students with the opportunity to practice critical and creative thinking skills.



G. What Software Is Available to Solve the LEPs?

Polymath. Polymath was developed by Professors Michael Cutlip and Mordechai Shacham. The Polymath software includes an ordinary differential equation (ODE) solver, a nonlinear equation solver, and nonlinear regression. As with previous editions of this book, Polymath is used to explore the example problems and to solve the homework problems. Polymath tutorials with screen shots are given on the CRE Web site Summary Notes in Chapter 1 and can also be accessed from the Home Page by going to Living Example Problems (LEPs) and then clicking on Polymath. Most chemical engineering departments in the United States have site licenses for Polymath. If your department does not have a site license and would like one, have your instructor e-mail the CACHE Corporation at cache@uts.cc.utexas.edu to learn how to obtain one. The LEPs need to be copied from the CRE Web site and pasted into the Polymath software. The Polymath software used in the examples in the text is available in most department computer labs in the United States. If you want to have Polymath on your personal laptop computer, you will need to purchase the program. An educational version of the software has been made available by Professors Cutlip and Shacham to students for \$20 for a 4-month license, \$30 for a 12-month license, or for \$39 for a perpetual-use license. Polymath 6.1 is compatible with Windows XP, Windows Vista, Windows 7, and Windows 8. See the Polymath Web site (<http://www.polymath-software.com>) to obtain the laptop version. A special Polymath Web site (<http://www.polymath-software.com/fogler>) has been set up for this book by Polymath authors Professors Cutlip and Shacham.

AspenTech. AspenTech is a process flow sheet simulator used in most senior chemical engineering design courses. It is now routinely introduced in earlier chemical engineering courses, such as thermodynamics, separations, and now in CRE. See the AspenTech Web site (<http://www.aspentech.com>) for more

information. Like Polymath, AspenTech site licenses are available in most chemical engineering departments in the United States. Four AspenTech simulation examples specific to CRE are provided on the CRE Web site with step-by-step tutorial screen shots.

As with Polymath programs, the input parameters in AspenTech can be varied to learn how they change the temperature and concentration profiles. Further details are given in Appendix D.

COMSOL Multiphysics. The COMSOL Multiphysics software is a partial differential equation solver that is used with Chapters 12 and 18 to view both axial and radial temperature and concentration profiles. For users of this text, COMSOL has provided a special Web site that includes a step-by-step tutorial, along with examples. See <http://www.comsol.com/ecre>. Further details are given in Appendix D.

Further details of these three software packages can be found in Appendix D.

H. Are There Other Web Site Resources?

FAQs. The Frequently Asked Questions (FAQs) page on the CRE Web site contains a compilation of questions collected over the years from undergraduate students taking reaction engineering.

Visual Encyclopedia of Equipment (<http://encyclopedia.che.engin.umich.edu>). This section was developed by Dr. Susan Montgomery at the University of Michigan. Here, a wealth of photographs and descriptions of real and ideal reactors are given. Students with visual, active, sensing, and intuitive learning styles of the Felder/Solomon Index will particularly benefit from this section.

Reactor Lab (<http://www.ReactorLab.net>). Developed by Professor Richard Herz at the University of California at San Diego, this interactive tool will allow students not only to test their comprehension of the CRE material, but also to explore different situations and combinations of reaction orders and types of reactions.

CRE Web Site. The CRE Web site (<http://www.umich.edu/~elements/5e/index.html>) will be used to update the text and identify typographical and other errors in the first and later printings of this text—available under **Updates and FAQs** on the CRE Web site home page. Additional material may also be added to include more solved problems, as well as additional Web Modules, which will also be found under **Updates and FAQs**.

I. How Can Critical Thinking and Creative Thinking Skills Be Enhanced?

I.1. Enhance Critical Thinking Skills

A third goal of this book is to enhance critical thinking skills. How does one enhance their critical thinking skills? Answer by learning how to ask the critical thinking questions in Table P-2 and carry out the actions in Table P-3. A number of homework problems have been included that are designed for this purpose. Socratic questioning is at the heart of critical thinking, and a number of homework problems draw from R. W. Paul's six types of Socratic questions,³ shown in Table P-2 and given in the expanded material on the Web site.

TABLE P-2 SIX TYPES OF SOCRATIC QUESTIONS USED IN CRITICAL THINKING

(1) <i>Questions for clarification:</i> Why do you say that? How does this relate to our discussion?	“Are you going to include diffusion in your mole balance equations?”
(2) <i>Questions that probe assumptions:</i> What could we assume instead? How can you verify or disprove that assumption?	“Why are you neglecting radial diffusion and including only axial diffusion?”
(3) <i>Questions that probe reasons and evidence:</i> What would be an example?	“Do you think that diffusion is responsible for the lower conversion?”
(4) <i>Questions about viewpoints and perspectives:</i> What would be an alternative?	“With all the bends in the pipe, from an industrial/practical perspective, do you think diffusion and dispersion will be large enough to affect the conversion?”
(5) <i>Questions that probe implications and consequences:</i> What generalizations can you make? What are the consequences of that assumption?	“How would the results be affected if you neglected diffusion?”
(6) <i>Questions about the question:</i> What was the point of this question? Why do you think I asked this question?	“Why do you think diffusion is important?”

It is important to know these six types and be able to apply them when investigating a problem such as “Is there a chance the reactor will run away and explode?” or “Why did the reactor explode?”

Critical thinking skills are like any skill, they must be practiced. Scheffer and Rubenfeld^{4,5} describe how to practice critical thinking skills using the activities, statements, and questions shown in Table P-3. The reader should try to practice using some or all of these actions every day, as well as asking the critical thinking questions in Table P-1 and on the Web site.

³ R. W. Paul, *Critical Thinking* (Santa Rosa, CA: Foundation for Critical Thinking, 1992).

⁴ Courtesy of B. K. Scheffer and M. G. Rubenfeld, “A Consensus Statement on Critical Thinking in Nursing,” *Journal of Nursing Education*, 39, 352–359 (2000).

⁵ Courtesy of B. K. Scheffer and M. G. Rubenfeld, “Critical Thinking: What Is It and How Do We Teach It?” *Current Issues in Nursing* (2001).

TABLE P-3 CRITICAL THINKING ACTIONS⁶

Analyzing: separating or breaking a whole into parts to discover their nature, function, and relationships “I studied it piece by piece.” “I sorted things out.”
Applying Standards: judging according to established personal, professional, or social rules or criteria “I judged it according to....”
Discriminating: recognizing differences and similarities among things or situations and distinguishing carefully as to category or rank “I rank ordered the various....” “I grouped things together.”
Information Seeking: searching for evidence, facts, or knowledge by identifying relevant sources and gathering objective, subjective, historical, and current data from those sources “I knew I needed to look up/study....” “I kept searching for data.”
Logical Reasoning: drawing inferences or conclusions that are supported in or justified by evidence “I deduced from the information that....” “My rationale for the conclusion was....”
Predicting: envisioning a plan and its consequences “I envisioned the outcome would be....” “I was prepared for....”
Transforming Knowledge: changing or converting the condition, nature, form, or function of concepts among contexts “I improved on the basics by....” “I wondered if that would fit the situation of”

I have found that the best way to develop and practice critical thinking skills is to use Tables P-2 and P-3 to help students write a question on any assigned homework problem and then to explain why the question involves critical thinking.

More information on critical thinking can be found on the CRE Web site in the section on Problem Solving (<http://www.umich.edu/~elements/5e/prob-solv/index.htm>).

I.2 Enhance Creative Thinking Skills

The fourth goal of this book is to help enhance creative thinking skills. This goal is achieved by using a number of problems that are open-ended to various degrees. With these, students can practice their creative skills by exploring the example problems, as outlined at the beginning of the home problems of each chapter, and by making up and solving an original problem. Problem P5-1 in the text gives some guidelines for developing original problems. A number of techniques that can aid students in practicing and enhancing their creativity

⁶ R. W. Paul, *Critical Thinking* (Santa Rosa, CA: Foundation for Critical Thinking, 1992); B. K. Scheffer and M. G. Rubenfeld, “A Consensus Statement on Critical Thinking in Nursing,” *Journal of Nursing Education*, 39, 352–359 (2000).

can be found in Fogler, LeBlanc, and Rizzo⁷ (and its companion Web site), *Strategies for Creative Problem Solving, Third Edition*. The Web site for that book can be accessed from the CRE Web site home page. We use these techniques, such as Osborn's checklist and de Bono's lateral thinking (which involves considering other people's views and responding to random stimulation) to answer add-on questions such as those in Table P-4.

TABLE P-4 PRACTICING CREATIVE THINKING

-
- (1) Brainstorm ideas to ask another question or suggest another calculation that can be made for this homework problem.
 - (2) Brainstorm ways you could work this homework problem incorrectly.
 - (3) Brainstorm ways to make this problem easier or more difficult or more exciting.
 - (4) Brainstorm a list of things you learned from working this homework problem and what you think the point of the problem is.
 - (5) Brainstorm the reasons why your calculations overpredicted the conversion that was measured when the reactor was put on stream. Assume you made no numerical errors in your calculations.
 - (6) "What if..." questions: The "What if..." questions are particularly effective when used with the *Living Example Problems*, where one varies the parameters to explore the problem and to carry out a sensitivity analysis. For example, *what if someone suggested that you should double the catalyst particle diameter, what would you say?*
-

One of the major goals at the undergraduate level is to bring students to the point where they can solve complex reaction problems, such as multiple reactions with heat effects, and then ask "What if . . . ?" questions and look for optimum operating conditions and unsafe operating conditions. The solution to one problem exemplifies this goal: the Manufacture of Styrene (Chapter 12, Problem P12-26_c). This problem is particularly interesting because two reactions are endothermic and one is exothermic.

- (1) Ethylbenzene \rightarrow Styrene + Hydrogen: Endothermic
- (2) Ethylbenzene \rightarrow Benzene + Ethylene: Endothermic
- (3) Ethylbenzene + Hydrogen \rightarrow Toluene + Methane: Exothermic

The student could get further practice in critical and creative thinking skills by adding any of the following exercises (x), (y), and (z) to any of the end-of-chapter homework problems.

- (x) How could you make this problem easier? More difficult?
- (y) Critique your answer by writing a critical thinking question.
- (z) Describe two ways you could work this problem incorrectly.



To summarize, it is this author's experience that both critical and creative thinking skills can be enhanced by using Tables P-2, P-3, and P-4 to extend any of the homework problems at the end of each chapter.

⁷ H. S. Fogler, S. E. LeBlanc, with B. Rizzo, *Strategies for Creative Problem Solving*, 3rd Ed. (Upper Saddle River, N.J.: Prentice Hall, 2014).

J. What's New in This Edition?

J.1 Pedagogy

This book maintains all the strengths of the fourth edition of *Elements of Chemical Reaction Engineering* by using algorithms that allow students to learn chemical reaction engineering through logic rather than memorization. It has the added strength of breaking down the material into smaller bites, as there are now 18 chapters to cover the same concepts as the 14 chapters in the fourth edition.

At the same time, this edition provides new resources that allow students to go beyond solving equations in order to get an intuitive feel and understanding of how reactors behave under different situations. This understanding is achieved through more than 80 interactive simulations (LEPs) provided on the Web site. The Web site has been greatly expanded to address the Felder/Solomon Inventory of Different Learning Styles⁸ through interactive Summary Notes and new and updated Interactive Computer Games (ICGs). For example, as discussed in Appendix I the Global Learner can get an overview of the chapter material from the Summary Notes; the Sequential Learner can use all the  hot buttons; and the active learner can interact with the ICGs and use the  hot buttons in the Summary Notes.

A new pedagogical concept is introduced in this text through expanded emphasis on the example problems. Here, the students simply load the Living Example Problems (LEPs) onto their computers and then explore the problems to obtain a deeper understanding of the implications and generalizations before working the homework problems for that chapter. This exploration helps students get an innate feel for reactor behavior and operation, as well as develop and practice their creative thinking skills.

To develop critical thinking skills, instructors can assign one of the new homework problems on troubleshooting, as well as ask the students to expand homework problems by asking a related question that involves critical thinking using Tables P-2 and P-3.

Creative thinking skills can be enhanced by exploring the example problems and asking “What if . . . ?” questions, by using one or more of the brainstorming exercises in Table P-4 to extend any of the homework problems, and by solving the open-ended problems. For example, in the case study on safety, students can use the LEP on the CRE Web site to carry out a postmortem analysis on the nitroaniline explosion in Example 13-2 to learn what would have happened if the cooling had failed for five minutes instead of ten minutes. To this end, a new feature in the text is an Analysis paragraph at the end of each example problem. Significant effort has been devoted to developing example and homework problems that foster critical and creative thinking.

⁸ <http://www.ncsu.edu/felder-public/ILSdir/styles.htm>

J.2 Content

The following areas have an increased emphasis in this new edition over previous CRE editions by including thorough example problems and homework problems:

1. Safety: Three industrial explosions are discussed and modeled.
 - a. Ammonium Nitrate CSTR Explosion (Chapters 12 and 13)
 - b. Nitroaniline Batch Reactor Runaway (Chapter 13)
 - c. T2 Laboratories Batch Reactor Runaway (Chapter 13)
 - d. Resources from SChE and CCPS (Chapter 12)
2. Solar Energy: Three examples of solar energy conversion are discussed.
 - a. Solar Chemical Reactions (Chapter 3)
 - b. Solar Thermal Reactors (Chapter 8)
 - c. Solar Catalytic Water Splitting (Chapter 10)
3. Alternative Fuels:
 - a. Production of Algae for Biomass (Chapter 9)
4. AspenTech: An AspenTech tutorial for chemical reaction engineering and four example problems are provided on the CRE Web site. The example problems are
 - a. Production of Ethylene from Ethane
 - b. The Pyrolysis of Benzene
 - c. Adiabatic Liquid Phase Isomerization of Normal Butane
 - d. Adiabatic Production of Acetic Anhydride

However, all intensive laws tend often to have exceptions. Very important concepts take orderly, responsible statements. Virtually all laws intrinsically are natural thoughts. General observations become laws under experimentation.

K. How Do I Say Thank You?

There are so many colleagues and students who contributed to this book that it would require another chapter to thank them all in an appropriate manner. I again acknowledge all my friends, students, and colleagues for their contributions to the fifth edition of *Elements of Chemical Reaction Engineering*. I would like to give special recognition as follows.

First of all, I am indebted to Ame and Catherine Vennema, whose gift of an endowed chair greatly facilitated the completion of this project. My colleague Dr. Nihat Gürmen coauthored the original Web site during the writing of the fourth edition of this book. He has been a wonderful colleague to work with. I also would like to thank University of Michigan undergraduate students Arthur Shih, Maria Quigley, and Brendan Kirchner, who worked on earlier versions of the Web site. Their hard work and suggestions are greatly appreciated. Ben Griessmann was instrumental in making everything come together for the Web site for the fifth edition, including converting the fourth edition's physical CD-ROM to online-only content for this new edition.

The many stimulating discussions on activation energy with Professor Michael Stamatakis in the Chemical Engineering Department at University College London are greatly appreciated. Michael B. Cutlip, coauthor of *Poly-math*, not only gave suggestions and a critical reading of many sections, but also, most importantly, provided continuous support and encouragement throughout the course of this project. Professor Chau-Chyun Chen provided two AspenTech examples. Ed Fontes at COMSOL Mutiphysic not only provided encouragement, but also provided a COMSOL Web site containing a tutorial with CRE examples. Bernard Goodwin and Laura Lewin, editors at Prentice Hall, were extremely encouraging, helpful, and supportive throughout. Julie Nahil, full-service production manager at Prentice Hall, was fantastic throughout. She provided encouragement, attention to detail, and a great sense of humor, which were greatly appreciated. Indian Institute of Technology (IIT) students Darshan Shah, Anamika Singh, and Sravya Jangareddy, along with Fan Zhang, a University of Michigan student, and Keyvan Edrisi from Swedish Royal Institute of Technology, not only participated in the preparation of the solutions manual, but along with Maithri Venkat worked on the Web site to place many of the LEPs in Wolfram. Richa Motwani from IIT Guwahati, and Gunish Handa and Prafful Bhansali from IIT Bombay, did an extraordinary job in proofreading the galley proofs of the manuscript and making helpful suggestions for changes as well as putting the solution manual in final form. Thank you to students Krittin Binabdullah, Apirak Hanpan, and Thanaphoom Khrutphisit from Chulalongkorn University in Bangkok, along with Ph.D. candidate Cláudio Vilas Bôas Fávero for help in meeting the final deadline for this manuscript.

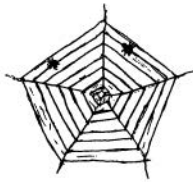
I very much appreciated the patience of all my Ph.D. students during the period in which this book was written, Michael Senra, Zhenyu Huang, Michael Hoepfner, Nasim Haji Akbari Balou, Claudio Vilas Boas Favero, and Mark Sheng Zheng. Mark helped proofread a number of chapters of the page proofs; Professor Michael Senra class-tested the draft version of the fifth edition, and he and his students gave many valuable suggestions to this edition. There are others I would like to thank for a variety of different reasons: David Bogle, Lee Brown, Brice Carnahan, John Chen, Stu Churchill, Rane Curl, Jim Duderstadt, Tom Edgar, John Falconer, Rich Felder, Asterios Gavriilidis, Joe Goddard, Jay Jorgenson, Costas Kravaris, Steve LeBlanc, Joe Martin, Susan Montgomery, Max Peters, Phil Savage, Johannes Schwank, Mordechai Shacham, Klaus Timmerhaus, Ron West, Jim Wilkes, June Wispelwey, Max, Joe (aka “Jofo”), Sophia, Nicolas, and to the Starbucks staff at Plymouth Road Mall, where most of my final editing of this book was accomplished.

Laura Bracken is very much a part of this book. I appreciate her excellent deciphering of equations and scribbles, her organization, her discovery of mistakes and inconsistencies, and her attention to detail in working with the galleys and proofs. Through all this was her ever-present wonderful disposition. Thanks, Radar!!

Finally, to my wife Janet, love and thanks. Not only did she type the first edition of this book—on a Royal Select typewriter!—she also was a sounding board for so many things in this edition. She was always willing to help with

the wording and sentence structure. For example, I often asked her, “Is this the correct phrase or word to use here?” or “Should I mention Jofostan here?” Jan also helped me learn that creativity also involves knowing what to leave out. Without her enormous help and support the project would never have been possible.

HSF
Ann Arbor, Michigan
November 2015



For updates and new and exciting applications, go to the Web site:

<http://www.umich.edu/~elements/5e/index.html>

For typographical errors, click on Updates & FAQ on the Home page to find

<http://www.umich.edu/~elements/5e/updates/index.html>

About the Author



H. Scott Fogler is the Ame and Catherine Vennema professor of chemical engineering and the Arthur F. Thurnau professor at the University of Michigan in Ann Arbor, and was the 2009 National President of the American Institute of Chemical Engineers, a 50,000-member organization. He received his B.S. from the University of Illinois and his M.S. and Ph.D. from the University of Colorado. He is also the author of the *Essentials of Chemical Reaction Engineering* and co-author, with Steven LeBlanc and Benjamin Rizzo, of *Strategies for Creative Problem Solving, Third Edition*.

Professor Fogler's research interests include flow and reaction in porous media, wax and asphaltene deposition, asphaltene flocculation kinetics, gelation kinetics, colloidal phenomena, and catalyzed dissolution. He has been research advisor to more than forty-five Ph.D. students and has more than two hundred thirty-five refereed publications in these areas. Fogler has chaired ASEE's Chemical Engineering Division, served as director of the American Institute of Chemical Engineers, and earned the Warren K. Lewis Award from AIChE for contributions to chemical engineering education. He also received the Chemical Manufacturers Association's National Catalyst Award and the 2010 Malcom E. Pruitt Award from the Council for Chemical Research (CCR). He is the recipient of 11 named lectureships and is associate editor of *Energy & Fuels*. On April 15th, 2016 Scott received a *doctor honoris causa* degree from the Universitat Rovira i Virgili, Tarragona, Spain.

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Mole Balances 1

The first step to knowledge
is to know that we are ignorant.

—Socrates (470–399 B.C.)

The Wide Wild World of Chemical Reaction Engineering

Chemical kinetics is the study of chemical reaction rates and reaction mechanisms. The study of chemical reaction engineering (CRE) combines the study of chemical kinetics with the reactors in which the reactions occur. Chemical kinetics and reactor design are at the heart of producing almost all industrial chemicals, such as the manufacture of phthalic anhydride shown in Figure 1-1. It is primarily a knowledge of chemical kinetics and reactor design that distinguishes the chemical engineer from other engineers. The selection of a reaction system that operates in the safest and most efficient manner can be the key to the economic success or failure of a chemical plant. For example, if a reaction system produces a large amount of undesirable product, subsequent purification and separation of the desired product could make the entire process economically unfeasible.

How is a chemical
engineer different
from other
engineers?

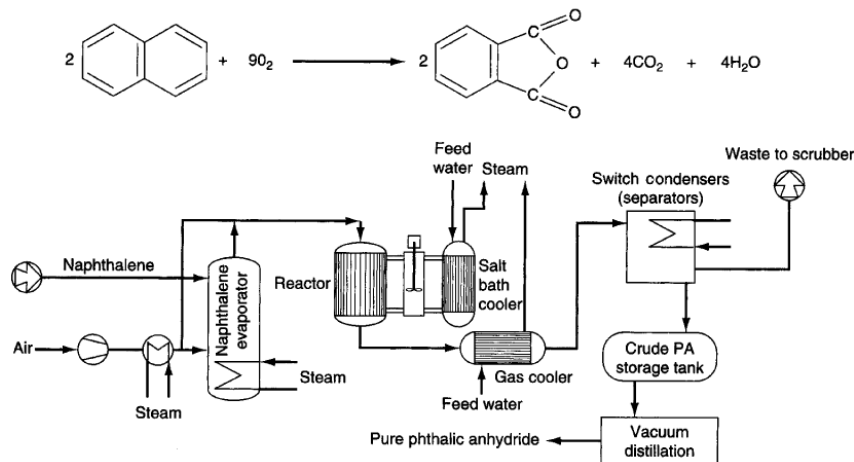
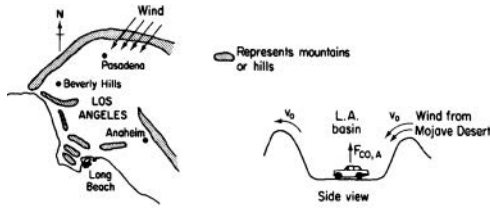
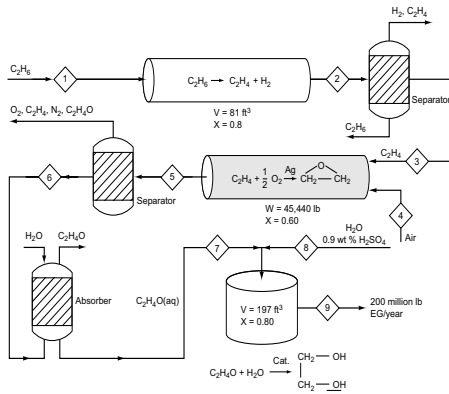


Figure 1-1 Manufacture of phthalic anhydride.

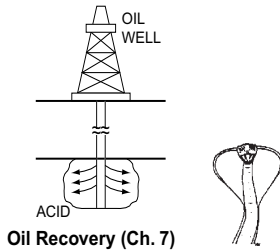
The chemical reaction engineering (CRE) principles learned here can also be applied in many areas, such as waste treatment, microelectronics, nanoparticles, and living systems, in addition to the more traditional areas of the manufacture of chemicals and pharmaceuticals. Some of the examples that illustrate the wide application of CRE principles in this book are shown in Figure 1-2. These examples include modeling smog in the Los Angeles (L.A.) basin (Chapter 1), the digestive system of a hippopotamus (Chapter 2 on the CRE Web site, www.umich.edu/~elements/5e/index.html), and molecular CRE (Chapter 3). Also shown are the manufacture of ethylene glycol (antifreeze), where three of the most common types of industrial reactors are used (Chapters 5 and 6), and the use of wetlands to degrade toxic chemicals (Chapter 7 on the CRE Web site). Other examples shown are the solid-liquid kinetics of acid-rock interactions to improve oil recovery (Chapter 7); pharmacokinetics of cobra bites (Chapter 8 Web Module); free-radical scavengers used in the design of motor oils (Chapter 9); enzyme kinetics (Chapter 9) and drug delivery pharmacokinetics (Chapter 9 on the CRE Web site); heat effects, runaway reactions, and plant safety (Chapters 11 through 13); and increasing the octane number of gasoline and the manufacture of computer chips (Chapter 10).



Smog (Ch. 1)



Chemical Plant for Ethylene Glycol (Ch. 5)



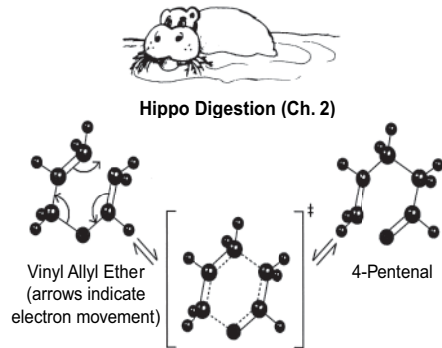
Oil Recovery (Ch. 7)



Effective Lubricant Design Scavenging Free Radicals
Lubricant Design (Ch. 9)

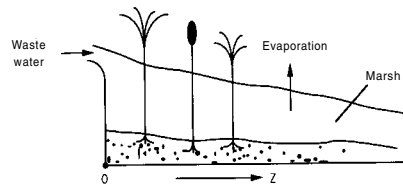
Pharmacokinetics of Cobra Bites
Multiple Reactions in a Batch (Body) Reactor

Cobra Bites (Ch. 8 on the CRE Web site)

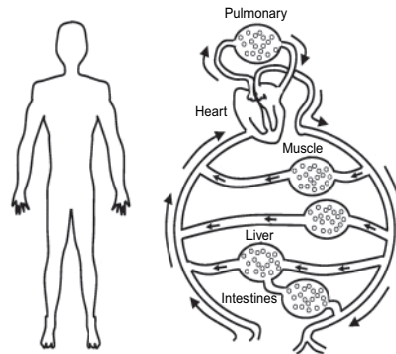


Transition State (dashed lines show transition state electron delocalization)

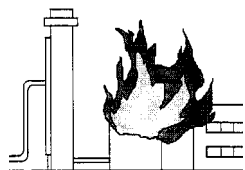
Molecular CRE (Ch. 3)



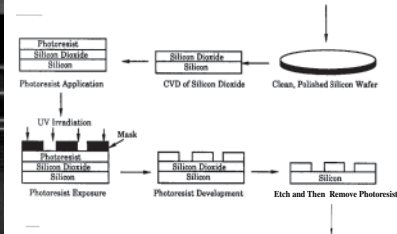
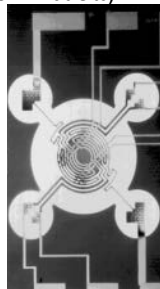
Wetlands Remediation of Pollutants (Ch. 7 on the CRE Web site)



Pharmacokinetics (Ch. 9 on the CRE Web site)



Nitroanaline Plant Explosion
Exothermic Reactions That Run Away
Plant Safety (Ch.11 to Ch.13)



Microelectronic Fabrication Steps (Ch. 10)

Figure 1-2 The wide world of CRE applications.

Overview—Chapter 1. This chapter develops the first building block of chemical reaction engineering, *mole balances*, which will be used continually throughout the text. After completing this chapter, the reader will be able to:

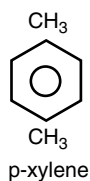
- Describe and define the rate of reaction
- Derive the general mole balance equation
- Apply the general mole balance equation to the four most common types of industrial reactors

Before entering into discussions of the conditions that affect chemical reaction rate mechanisms and reactor design, it is necessary to account for the various chemical species entering and leaving a reaction system. This accounting process is achieved through overall mole balances on individual species in the reacting system. In this chapter, we develop a general mole balance that can be applied to any species (usually a chemical compound) entering, leaving, and/or remaining within the reaction system volume. After defining the rate of reaction, $-r_A$, we show how the general balance equation may be used to develop a preliminary form of the design equations of the most common industrial reactors:

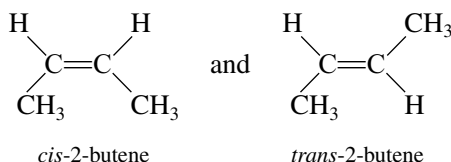
- Batch Reactor (BR)
- Continuous-Stirred Tank Reactor (CSTR)
- Plug-Flow Reactor (PFR)
- Packed-Bed Reactor (PBR)

In developing these equations, the assumptions pertaining to the modeling of each type of reactor are delineated. Finally, a brief summary and series of short review questions are given at the end of the chapter.

1.1 The Rate of Reaction, $-r_A$

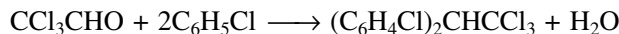


The rate of reaction tells us how fast a number of moles of one chemical species are being consumed to form another chemical species. The term *chemical species* refers to any chemical component or element with a given *identity*. The identity of a chemical species is determined by the *kind*, *number*, and *configuration* of that species' atoms. For example, the species para-xylene is made up of a fixed number of specific atoms in a definite molecular arrangement or configuration. The structure shown illustrates the kind, number, and configuration of atoms on a molecular level. Even though two chemical compounds have exactly the same kind and number of atoms of each element, they could still be different species because of different configurations. For example, 2-butene has four carbon atoms and eight hydrogen atoms; however, the atoms in this compound can form two different arrangements.



To summarize this point, we say that a given number of molecules (i.e., moles) of a particular chemical species have reacted or disappeared when the molecules have lost their chemical identity.

The rate at which a given chemical reaction proceeds can be expressed in several ways. To illustrate, consider the reaction of chloral and chlorobenzene to produce the banned insecticide DDT (dichlorodiphenyl-trichloroethane) in the presence of fuming sulfuric acid.



Letting the symbol A represent chloral, B be chlorobenzene, C be DDT, and D be H_2O , we obtain



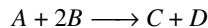
The numerical value of the rate of disappearance of reactant A, $-r_A$, is a positive number.

What is $-r_A$?

The rate of reaction, $-r_A$, is the number of moles of A (e.g., chloral) reacting (disappearing) per unit time per unit volume ($\text{mol}/\text{dm}^3\cdot\text{s}$).

Example 1-1

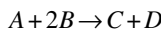
Chloral is being consumed at a rate of 10 moles per second per m^3 when reacting with chlorobenzene to form DDT and water in the reaction described above. In symbol form, the reaction is written as



Write the rates of disappearance and formation (i.e., generation) for each species in this reaction.

Solution

- (a) *Chloral*[A]: The rate of reaction of chloral [A] ($-r_A$) is given as $10 \text{ mol}/\text{m}^3\cdot\text{s}$
 Rate of disappearance of A = $-r_A = 10 \text{ mol}/\text{m}^3\cdot\text{s}$
 Rate of formation of A = $r_A = -10 \text{ mol}/\text{m}^3\cdot\text{s}$
- (b) *Chlorobenzene*[B]: For every mole of chloral that disappears, two moles of chlorobenzene [B] also disappear.
 Rate of disappearance of B = $-r_B = 20 \text{ mol}/\text{m}^3\cdot\text{s}$
 Rate of formation of B = $r_B = -20 \text{ mol}/\text{m}^3\cdot\text{s}$
- (c) *DDT*[C]: For every mole of chloral that disappears, one mole of DDT [C] appears.
 Rate of formation of C = $r_C = 10 \text{ mol}/\text{m}^3\cdot\text{s}$
 Rate of disappearance of C = $-r_C = -10 \text{ mol}/\text{m}^3\cdot\text{s}$
- (d) *Water*[D]: Same relationship to chloral as the relationship to DDT
 Rate of formation of D = $r_D = 10 \text{ mol}/\text{m}^3\cdot\text{s}$
 Rate of disappearance of D = $-r_D = -10 \text{ mol}/\text{m}^3\cdot\text{s}$



The convention

$-r_A = 10 \text{ mol A/m}^3\cdot\text{s}$
$r_A = -10 \text{ mol A/m}^3\cdot\text{s}$
$-r_B = 20 \text{ mol B/m}^3\cdot\text{s}$
$r_B = -20 \text{ mol B/m}^3\cdot\text{s}$
$r_C = 10 \text{ mol C/m}^3\cdot\text{s}$

Analysis: The purpose of this example is to better understand the convention for the rate of reaction. The symbol r_j is the rate of formation (generation) of species j . If species j is a reactant, the numerical value of r_j will be a negative number. If species j is a product, then r_j will be a positive number. The rate of reaction, $-r_A$, is the rate of disappearance of reactant A and must be a positive number. A mnemonic relationship to help remember how to obtain relative rates of reaction of A to B, etc., is given by Equation (3-1) on page 71.

In Chapter 3, we will delineate the prescribed relationship between the rate of formation of one species, r_j (e.g., DDT [C]), and the rate of disappearance of another species, $-r_i$ (e.g., chlorobenzene [B]), in a chemical reaction.

Heterogeneous reactions involve more than one phase. In heterogeneous reaction systems, the rate of reaction is usually expressed in measures other than volume, such as reaction surface area or catalyst weight. For a gas-solid catalytic reaction, the gas molecules must interact with the solid catalyst surface for the reaction to take place, as described in Chapter 10.

What is $-r'_A$? The dimensions of this heterogeneous reaction rate, $-r'_A$ (prime), are the number of moles of A reacting per unit time per unit mass of catalyst (mol/s·g catalyst).

Definition of r_j Most of the introductory discussions on chemical reaction engineering in this book focus on homogeneous systems, in which case we simply say that r_j is the rate of formation of species j per unit volume. It is the number of moles of species j generated per unit volume per unit time.

We can say four things about the reaction rate r_j . The reaction rate law for r_j is

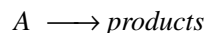
The rate law does not depend on the type of reactor used!!

- The rate of formation of species j (mole/time/volume)
- An algebraic equation
- Independent of the type of reactor (e.g., batch or continuous flow) in which the reaction is carried out
- Solely a function of the properties of the reacting materials and reaction conditions (e.g., species concentration, temperature, pressure, or type of catalyst, if any) at a point in the system

What is $-r_A$ a function of?

However, because the properties and reaction conditions of the reacting materials may vary with position in a chemical reactor, r_j can in turn be a function of position and can vary from point to point in the system.

The chemical reaction rate law is essentially an algebraic equation involving concentration, not a differential equation.¹ For example, the algebraic form of the rate law for $-r_A$ for the reaction



may be a linear function of concentration,

$$-r_A = kC_A \quad (1-1)$$

or, as shown in Chapter 3, it may be some other algebraic function of concentration, such as

¹ For further elaboration on this point, see *Chem. Eng. Sci.*, 25, 337 (1970); B. L. Crynes and H. S. Fogler, eds., *AIChE Modular Instruction Series E: Kinetics*, 1, 1 (New York: AIChE, 1981); and R. L. Kabel, "Rates," *Chem. Eng. Commun.*, 9, 15 (1981).

$$-r_A = kC_A^2 \quad (1-2)$$

or

The rate law is an algebraic equation.

$$-r_A = \frac{k_1 C_A}{1 + k_2 C_A}$$

The convention

For a given reaction, the particular concentration dependence that the rate law follows (i.e., $-r_A = kC_A$ or $-r_A = kC_A^2$ or ...) must be determined from *experimental observation*. Equation (1-2) states that the rate of disappearance of A is equal to a rate constant k (which is a function of temperature) times the square of the concentration of A. As noted earlier, by convention, r_A is the rate of formation of A; consequently, $-r_A$ is the rate of disappearance of A. Throughout this book, the phrase *rate of generation* means exactly the same as the phrase *rate of formation*, and these phrases are used interchangeably.

1.2 The General Mole Balance Equation

To perform a mole balance on any system, the system boundaries must first be specified. The volume enclosed by these boundaries is referred to as the *system volume*. We shall perform a mole balance on species j in a system volume, where species j represents the particular chemical species of interest, such as water or NaOH (Figure 1-3).

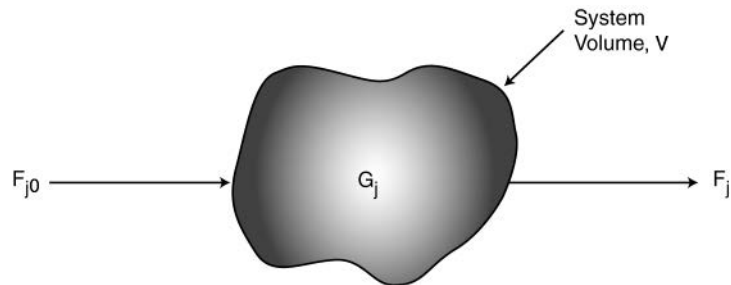


Figure 1-3 Mole balance on species j in a system volume, V .

A mole balance on species j at any instant in time, t , yields the following equation:

	$\left[\begin{array}{c} \text{Rate of flow} \\ \text{of } j \text{ into} \\ \text{the system} \\ \text{(moles/time)} \end{array} \right]$	-	$\left[\begin{array}{c} \text{Rate of flow} \\ \text{of } j \text{ out of} \\ \text{the system} \\ \text{(moles/time)} \end{array} \right]$	+	$\left[\begin{array}{c} \text{Rate of generation} \\ \text{of } j \text{ by chemical} \\ \text{reaction within} \\ \text{the system} \\ \text{(moles/time)} \end{array} \right]$	=	$\left[\begin{array}{c} \text{Rate of} \\ \text{accumulation} \\ \text{of } j \text{ within} \\ \text{the system} \\ \text{(moles/time)} \end{array} \right]$
Mole balance	In		Out		Generation		Accumulation
	F_{j0}		F_j		G_j		$= \frac{dN_j}{dt}$ (1-3)

In this equation, N_j represents the number of moles of species j in the system at time t . If all the system variables (e.g., temperature, catalytic activity, and concentration of the chemical species) are spatially uniform throughout the system volume, the rate of generation of species j , G_j , is just the product of the reaction volume, V , and the rate of formation of species j , r_j .

$$G_j = r_j \cdot V$$

$$\frac{\text{moles}}{\text{time}} = \frac{\text{moles}}{\text{time} \cdot \text{volume}} \cdot \text{volume}$$

Now suppose that the rate of formation of species j for the reaction varies with position in the system volume. That is, it has a value r_{j1} at location 1, which is surrounded by a small volume, ΔV_1 , within which the rate is uniform; similarly, the reaction rate has a value r_{j2} at location 2 and an associated volume, ΔV_2 , and so on (Figure 1-4).

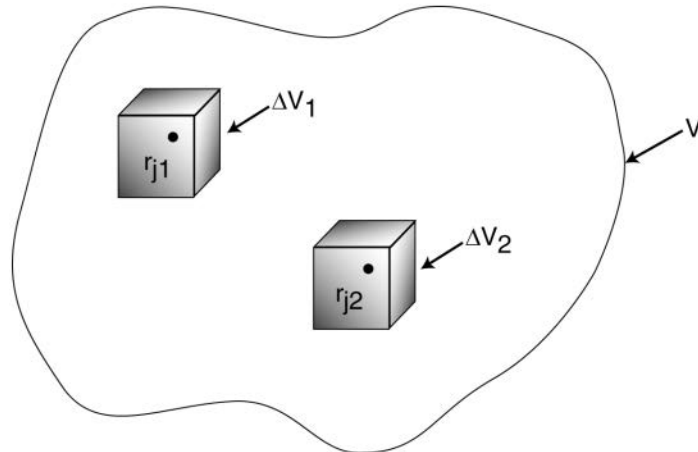


Figure 1-4 Dividing up the system volume, V .

The rate of generation, ΔG_{j1} , in terms of r_{j1} and subvolume ΔV_1 , is

$$\Delta G_{j1} = r_{j1} \Delta V_1$$

Similar expressions can be written for ΔG_{j2} and the other system subvolumes, ΔV_i . The total rate of generation within the system volume is the sum of all the rates of generation in each of the subvolumes. If the total system volume is divided into M subvolumes, the total rate of generation is

$$G_j = \sum_{i=1}^M \Delta G_{ji} = \sum_{i=1}^M r_{ji} \Delta V_i$$

By taking the appropriate limits (i.e., let $M \rightarrow \infty$ and $\Delta V \rightarrow 0$) and making use of the definition of an integral, we can rewrite the foregoing equation in the form

$$G_j = \int^V r_j dV$$

From this equation, we see that r_j will be an indirect function of position, since the properties of the reacting materials and reaction conditions (e.g., concentration, temperature) can have different values at different locations in the reactor volume.

We now replace G_j in Equation (1-3)

$$F_{j0} - F_j + G_j = \frac{dN_j}{dt} \quad (1-3)$$

by its integral form to yield a form of the general mole balance equation for any chemical species j that is entering, leaving, reacting, and/or accumulating within any system volume V .

This is a basic equation for chemical reaction engineering.

$$F_{j0} - F_j + \int^V r_j dV = \frac{dN_j}{dt} \quad (1-4)$$

From this general mole balance equation, we can develop the design equations for the various types of industrial reactors: batch, semibatch, and continuous-flow. Upon evaluation of these equations, we can determine the time (batch) or reactor volume (continuous-flow) necessary to convert a specified amount of the reactants into products.

1.3 Batch Reactors (BRs)

When is a batch reactor used?

A batch reactor is used for small-scale operation, for testing new processes that have not been fully developed, for the manufacture of expensive products, and for processes that are difficult to convert to continuous operations. The reactor can be charged (i.e., filled) through the holes at the top (see Figure 1-5(a)). The batch reactor has the advantage of high conversions that can be obtained by leaving the reactant in the reactor for long periods of time, but it also has the disadvantages of high labor costs per batch, the variability of products from batch to batch, and the difficulty of large-scale production (see Industrial Reactor Photos in *Professional Reference Shelf [PRS]* on the CRE Web site, www.umich.edu/~elements/5e/index.html).



Reference Shelf

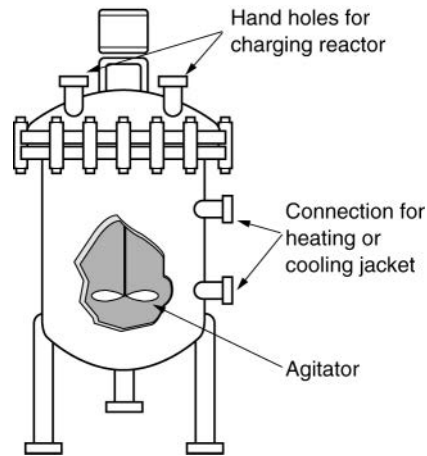


Figure 1-5(a) Simple batch homogeneous batch reactor (BR). [Excerpted by special permission from *Chem. Eng.*, 63(10), 211 (Oct. 1956). Copyright 1956 by McGraw-Hill, Inc., New York, NY 10020.]



Figure 1-5(b) Batch reactor mixing patterns. Further descriptions and photos of the batch reactors can be found in both the *Visual Encyclopedia of Equipment* and in the *Professional Reference Shelf* on the CRE Web site.

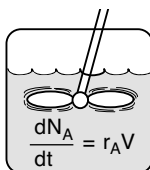
A batch reactor has neither inflow nor outflow of reactants or products while the reaction is being carried out: $F_{j0} = F_j = 0$. The resulting general mole balance on species j is

$$\frac{dN_j}{dt} = \int^V r_j dV$$

If the reaction mixture is perfectly mixed (Figure 1-5(b)) so that there is no variation in the rate of reaction throughout the reactor volume, we can take r_j out of the integral, integrate, and write the mole balance in the form

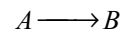
Perfect mixing

$$\boxed{\frac{dN_j}{dt} = r_j V} \quad (1-5)$$



Batch Reactor

Let's consider the isomerization of species A in a batch reactor



As the reaction proceeds, the number of moles of A decreases and the number of moles of B increases, as shown in Figure 1-6.

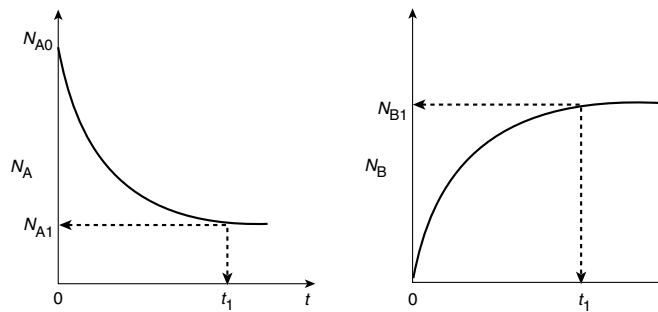


Figure 1-6 Mole-time trajectories.

We might ask what time, t_1 , is necessary to reduce the initial number of moles from N_{A0} to a final desired number N_{A1} . Applying Equation (1-5) to the isomerization

$$\frac{dN_A}{dt} = r_A V$$

rearranging,

$$dt = \frac{dN_A}{r_A V}$$

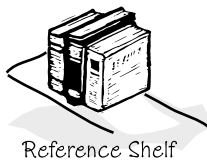
and integrating with limits that at $t = 0$, then $N_A = N_{A0}$, and at $t = t_1$, then $N_A = N_{A1}$, we obtain

$$t_1 = \int_{N_{A1}}^{N_{A0}} \frac{dN_A}{-r_A V} \quad (1-6)$$

This equation is the integral form of the mole balance on a batch reactor. It gives the time, t_1 , necessary to reduce the number of moles from N_{A0} to N_{A1} and also to form N_{B1} moles of B.

1.4 Continuous-Flow Reactors

Continuous-flow reactors are almost always operated at steady state. We will consider three types: the *continuous-stirred tank reactor* (CSTR), the *plug-flow reactor* (PFR), and the *packed-bed reactor* (PBR). Detailed physical descriptions of these reactors can be found in both the *Professional Reference Shelf* (PRS) for Chapter 1 and in the *Visual Encyclopedia of Equipment*, encyclopedia.che.engin.umich.edu, and on the CRE Web site.



Reference Shelf

1.4.1 Continuous-Stirred Tank Reactor (CSTR)

A type of reactor commonly used in industrial processing is the stirred tank operated continuously (Figure 1-7). It is referred to as the *continuous-stirred tank reactor* (CSTR) or *vat*, or *backmix reactor*, and is primarily used for

What is a CSTR used for?



Figure 1-7(a) CSTR/batch reactor. (Photo courtesy of Pfaunder, Inc.)

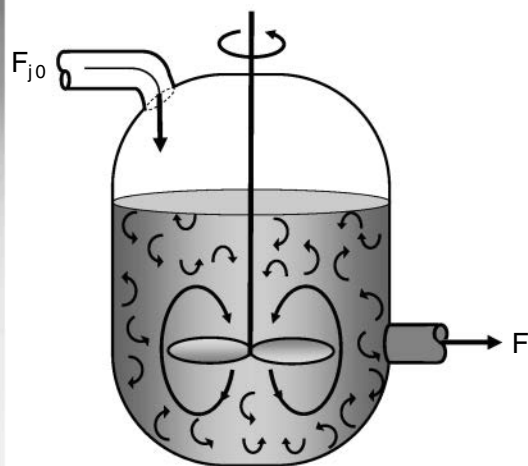


Figure 1-7(b) CSTR mixing patterns. Also see the *Visual Encyclopedia of Equipment* on the CRE Web site.

liquid-phase reactions. It is normally operated **at steady state** and is assumed to be **perfectly mixed**; consequently, there is no time dependence or position dependence of the temperature, concentration, or reaction rate inside the CSTR. That is, every variable is the same at every point inside the reactor. Because the temperature and concentration are identical everywhere within the reaction vessel, they are the same at the *exit point* as they are elsewhere in the tank. Thus, the temperature and concentration in the exit stream are modeled as being the same as those inside the reactor. In systems where mixing is highly nonideal, the well-mixed model is inadequate, and we must resort to other modeling techniques, such as residence time distributions, to obtain meaningful results. This topic of nonideal mixing is discussed in Chapters 16, 17, and 18 on nonideal reactors.

When the general mole balance equation

$$F_{j0} - F_j + \int_V r_j dV = \frac{dN_j}{dt} \quad (1-4)$$

is applied to a CSTR operated at steady state (i.e., conditions do not change with time),

$$\frac{dN_j}{dt} = 0$$

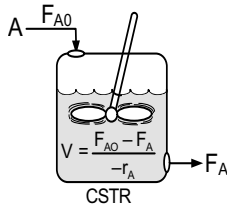
in which there are no spatial variations in the rate of reaction (i.e., perfect mixing),

The ideal CSTR is assumed to be perfectly mixed.

$$\int^V r_j dV = Vr_j$$

it takes the familiar form known as the *design equation* for a CSTR

$$V = \frac{F_{j0} - F_j}{-r_j} \quad (1-7)$$



The CSTR design equation gives the reactor volume V necessary to reduce the entering flow rate of species j from F_{j0} to the exit flow rate F_j , when species j is disappearing at a rate of $-r_j$. We note that the CSTR is modeled such that the conditions in the exit stream (e.g., concentration and temperature) **are identical** to those in the tank. The molar flow rate F_j is just the product of the concentration of species j and the volumetric flow rate v

$$F_j = C_j \cdot v$$

$$\frac{\text{moles}}{\text{time}} = \frac{\text{moles}}{\text{volume}} \cdot \frac{\text{volume}}{\text{time}} \quad (1-8)$$

Similarly, for the entrance molar flow rate we have $F_{j0} = C_{j0} \cdot v_0$. Consequently, we can substitute for F_{j0} and F_j into Equation (1-7) to write a balance on species A as

$$V = \frac{v_0 C_{A0} - v C_A}{-r_A} \quad (1-9)$$

The ideal CSTR mole balance equation is an algebraic equation, not a differential equation.

1.4.2 Tubular Reactor

In addition to the CSTR and batch reactors, another type of reactor commonly used in industry is the *tubular reactor*. It consists of a cylindrical pipe and is normally operated at steady state, as is the CSTR. Tubular reactors are used most often for gas-phase reactions. A schematic and a photograph of industrial tubular reactors are shown in Figure 1-8.

In the tubular reactor, the reactants are continually consumed as they flow down the length of the reactor. In modeling the tubular reactor, we assume that the concentration varies continuously in the axial direction through the reactor. Consequently, the reaction rate, which is a function of concentration for all but zero-order reactions, will also vary axially. For the purposes of the material presented here, we consider systems in which the flow field may be modeled by that of a plug-flow profile (e.g., uniform velocity as

When is a tubular reactor most often used?

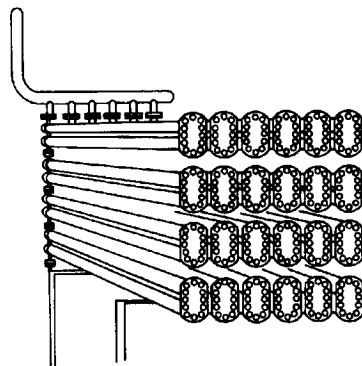


Figure 1-8(a) Tubular reactor schematic. Longitudinal tubular reactor. [Excerpted by special permission from *Chem. Eng.*, 63(10), 211 (Oct. 1956). Copyright 1956 by McGraw-Hill, Inc., New York, NY 10020.]

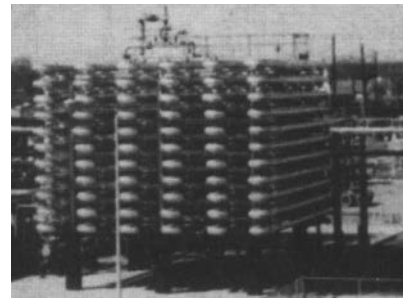


Figure 1-8(b) Tubular reactor photo. Tubular reactor for production of Dimersol G. (Photo courtesy of Editions Techniq Institut français du pétrole.)

in turbulent flow), as shown in Figure 1-9. That is, there is no radial variation in reaction rate, and the reactor is referred to as a *plug-flow reactor* (PFR). (The laminar-flow reactor is discussed in Chapters 16 through 18 on nonideal reactors.)

Also see PRS and *Visual Encyclopedia of Equipment*.

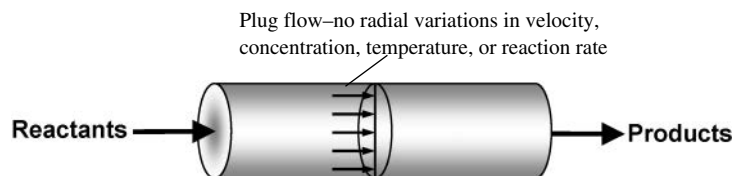


Figure 1-9 Plug-flow tubular reactor.

The general mole balance equation is given by Equation (1-4)

$$F_{j0} - F_j + \int^V r_j dV = \frac{dN_j}{dt} \quad (1-4)$$

The equation we will use to design PFRs at steady state can be developed in two ways: (1) directly from Equation (1-4) by differentiating with respect to volume V , and then rearranging the result or (2) from a mole balance on species j in a differential segment of the reactor volume ΔV . Let's choose the second way to arrive at the differential form of the PFR mole balance. The differential volume, ΔV , shown in Figure 1-10, will be chosen sufficiently small such that there are no spatial variations in reaction rate within this volume. Thus the generation term, ΔG_j , is

$$\Delta G_j = \int^{\Delta V} r_j dV = r_j \Delta V$$

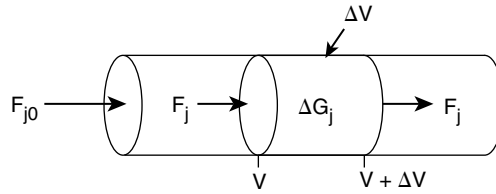


Figure 1-10 Mole balance on species j in volume ΔV .

$\left[\begin{array}{l} \text{Molar flow} \\ \text{rate of species } j \\ \text{In at } V \\ \text{moles/time} \end{array} \right]$	-	$\left[\begin{array}{l} \text{Molar flow} \\ \text{rate of species } j \\ \text{Out at } (V + \Delta V) \\ \text{moles/time} \end{array} \right]$	+	$\left[\begin{array}{l} \text{Molar rate of} \\ \text{Generation} \\ \text{of species } j \\ \text{within } \Delta V \\ \text{moles/time} \end{array} \right]$	=	$\left[\begin{array}{l} \text{Molar rate of} \\ \text{Accumulation} \\ \text{of species } j \\ \text{within } \Delta V \\ \text{moles/time} \end{array} \right]$
In		Out		Generation		Accumulation
$F_j _V$		$F_j _{V+\Delta V}$		$r_j \Delta V$		$= 0$ (1-10)

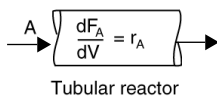
Dividing by ΔV and rearranging

$$\left[\frac{F_j|_{V+\Delta V} - F_j|_V}{\Delta V} \right] = r_j$$

the term in brackets resembles the definition of a derivative

$$\lim_{\Delta x \rightarrow 0} \left[\frac{f(x + \Delta x) - f(x)}{\Delta x} \right] = \frac{df}{dx}$$

Taking the limit as ΔV approaches zero, we obtain the differential form of steady state mole balance on a PFR



$$\boxed{\frac{dF_j}{dV} = r_j} \quad (1-11)$$

We could have made the cylindrical reactor on which we carried out our mole balance an irregularly shaped reactor, such as the one shown in Figure 1-11 for reactant species A.

However, we see that by applying Equation (1-10), the result would yield the same equation (i.e., Equation (1-11)). For species A, the mole balance is

$$\boxed{\frac{dF_A}{dV} = r_A} \quad (1-12)$$

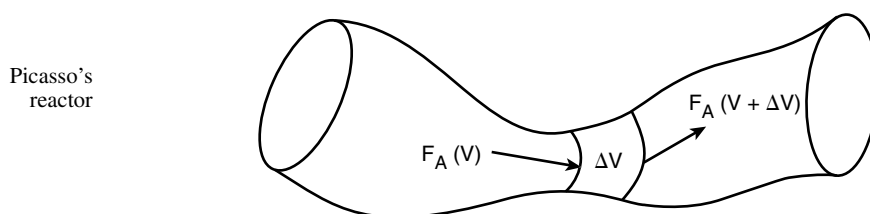


Figure 1-11 Pablo Picasso's reactor.

Consequently, we see that Equation (1-11) applies equally well to our model of tubular reactors of variable and constant cross-sectional area, although it is doubtful that one would find a reactor of the shape shown in 1-11 unless it were designed by Pablo Picasso.

The conclusion drawn from the application of the design equation to Picasso's reactor is an important one: the degree of completion of a reaction achieved in an ideal plug-flow reactor (PFR) does not depend on its shape, only on its total volume.

Again consider the isomerization $A \rightarrow B$, this time in a PFR. As the reactants proceed down the reactor, A is consumed by chemical reaction and B is produced. Consequently, the molar flow rate F_A decreases, while F_B increases as the reactor volume V increases, as shown in Figure 1-12.

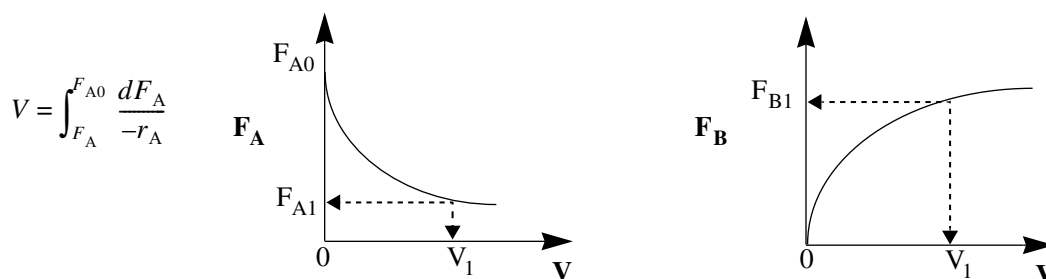


Figure 1-12 Profiles of molar flow rates in a PFR.

We now ask what is the reactor volume V_1 necessary to reduce the entering molar flow rate of A from F_{A0} to F_{A1} . Rearranging Equation (1-12) in the form

$$dV = \frac{dF_A}{r_A}$$

and integrating with limits at $V = 0$, then $F_A = F_{A0}$, and at $V = V_1$, then $F_A = F_{A1}$

$$V_1 = \int_{F_{A0}}^{F_{A1}} \frac{dF_A}{r_A} = \int_{F_{A1}}^{F_{A0}} \frac{dF_A}{-r_A} \quad (1-13)$$

V_1 is the volume necessary to reduce the entering molar flow rate F_{A0} to some specified value F_{A1} and also the volume necessary to produce a molar flow rate of B of F_{B1} .

1.4.3 Packed-Bed Reactor (PBR)

The principal difference between reactor design calculations involving homogeneous reactions and those involving fluid-solid heterogeneous reactions is that for the latter, the reaction takes place on the surface of the catalyst (see Chapter 10). The greater the mass of a given catalyst, the greater the reactive surface area. Consequently, the reaction rate is based on mass of solid catalyst, W , rather than on reactor volume, V . For a fluid–solid heterogeneous system, the rate of reaction of a species A is defined as

$$-r'_A = \text{mol A reacting}/(\text{time} \times \text{mass of catalyst})$$

The mass of solid catalyst is used because the amount of catalyst is what is important to the rate of product formation. We note that by multiplying the heterogeneous reaction rate, $-r'_A$, by the bulk catalyst density, $\rho_b \left(\frac{\text{mass}}{\text{volume}} \right)$, we can obtain the homogeneous reaction rate, $-r_A$

$$-r_A = \rho_b (-r'_A)$$

$$\left(\frac{\text{mol}}{\text{dm}^3 \cdot \text{s}} \right) = \left(\frac{\text{g}}{\text{dm}^3} \right) \left(\frac{\text{mol}}{\text{g} \cdot \text{s}} \right)$$

The reactor volume that contains the catalyst is of secondary significance. Figure 1-13 shows a schematic of an industrial catalytic reactor with vertical tubes packed with solid catalyst.

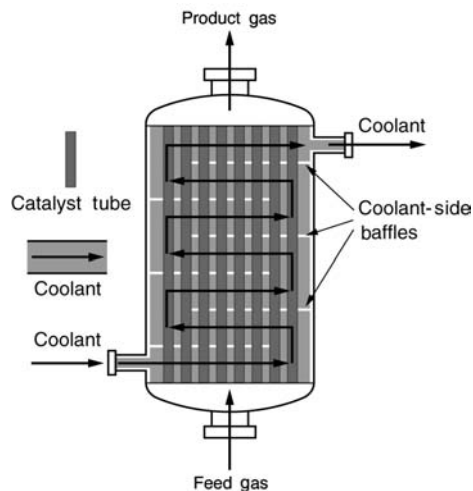


Figure 1-13 Longitudinal catalytic packed-bed reactor. [From Cropley, *American Institute of Chemical Engineers*, 86(2), 34 (1990). Reproduced with permission of the American Institute of Chemical Engineers, Copyright © 1990 AIChE. All rights reserved.]

PBR
Mole Balance

In the three idealized types of reactors just discussed (the perfectly mixed batch reactor, the plug-flow tubular reactor [PFR]), and the perfectly mixed continuous-stirred tank reactor [CSTR]), the design equations (i.e., mole balances) were developed based on reactor volume. The derivation of the design equation for a packed-bed catalytic reactor (PBR) will be carried out in a manner analogous to the development of the tubular design equation. To accomplish this derivation, we simply replace the volume coordinate in Equation (1-10) with the catalyst mass (i.e., weight) coordinate W (Figure 1-14).

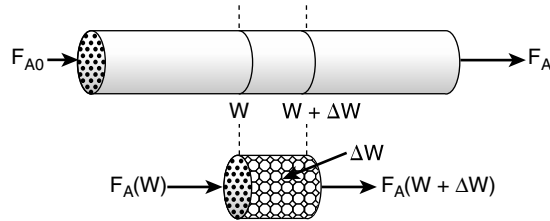


Figure 1-14 Packed-bed reactor schematic.

As with the PFR, the PBR is assumed to have no radial gradients in concentration, temperature, or reaction rate. The generalized mole balance on species A over catalyst weight ΔW results in the equation

$$\begin{aligned} \text{In} - \text{Out} + \text{Generation} &= \text{Accumulation} \\ F_{A|W} - F_{A|(W+\Delta W)} + r'_A \Delta W &= 0 \end{aligned} \quad (1-14)$$

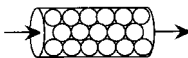
The dimensions of the generation term in Equation (1-14) are

$$(r'_A) \Delta W \equiv \frac{\text{moles A}}{(\text{time})(\text{mass of catalyst})} \cdot (\text{mass of catalyst}) \equiv \frac{\text{moles A}}{\text{time}}$$

which are, as expected, the same dimensions of the molar flow rate F_A . After dividing by ΔW and taking the limit as $\Delta W \rightarrow 0$, we arrive at the differential form of the mole balance for a packed-bed reactor

Use the differential form of design equation for catalyst decay and pressure drop.

$$\boxed{\frac{dF_A}{dW} = r'_A} \quad (1-15)$$



When pressure drop through the reactor (see Section 5.5) and catalyst decay (see Section 10.7 on the CRE Web site Chapter 10) are neglected, the integral form of the packed-catalyst-bed design equation can be used to calculate the catalyst weight

You can use the integral form **only** when there is no ΔP and no catalyst decay.

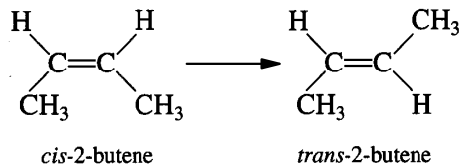
$$W = \int_{F_{A0}}^{F_A} \frac{dF_A}{r'_A} = \int_{F_A}^{F_{A0}} \frac{dF_A}{-r'_A} \quad (1-16)$$

W is the catalyst weight necessary to reduce the entering molar flow rate of species A, F_{A0} , down to a flow rate F_A .

For some insight into things to come, consider the following example of how one can use the tubular reactor design in Equation (1-11).

Example 1-2 How Large Is It?

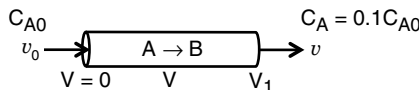
Consider the liquid phase *cis* – *trans* isomerization of 2-butene



which we will write symbolically as



The reaction is first order in A ($-r_A = kC_A$) and is carried out in a tubular reactor in which the volumetric flow rate, v , is constant, i.e., $v = v_0$.



1. Sketch the concentration profile.
2. Derive an equation relating the reactor volume to the entering and exiting concentrations of A, the rate constant k , and the volumetric flow rate v_0 .
3. Determine the reactor volume, V_1 , necessary to reduce the exiting concentration to 10% of the entering concentration, i.e., $C_A = 0.1C_{A0}$, when the volumetric flow rate is 10 dm³/min (i.e., liters/min) and the specific reaction rate, k , is 0.23 min⁻¹.

Solution

1. Sketch C_A as a function of V .

Species A is consumed as we move down the reactor, and as a result, both the molar flow rate of A and the concentration of A will decrease as we move. Because the volumetric flow rate is constant, $v = v_0$, one can use Equation (1-8) to obtain the concentration of A, $C_A = F_A/v_0$, and then by comparison with the Figure 1-12 plot, obtain the concentration of A as a function of reactor volume, as shown in Figure E1-2.1.

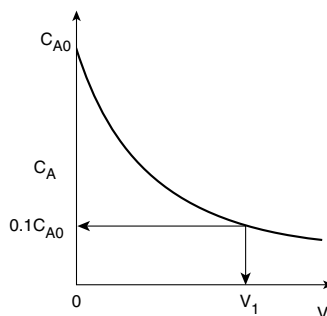


Figure E1-2.1 Concentration profile.

2. Derive an equation relating V , v_0 , k , C_{A0} , and C_A .

For a tubular reactor, the mole balance on species A ($j = A$) was shown to be given by Equation (1-11). Then for species A ($j = A$)

$$\text{Mole Balance:} \quad \frac{dF_A}{dV} = r_A \quad (1-12)$$

For a first-order reaction, the rate law (discussed in Chapter 3) is

$$\text{Rate Law:} \quad -r_A = kC_A \quad (E1-2.1)$$

Because the volumetric flow rate, v , is constant ($v = v_0$), as it is for most all liquid-phase reactions,

$$\frac{dF_A}{dV} = \frac{d(C_A v)}{dV} = \frac{d(C_A v_0)}{dV} = v_0 \frac{dC_A}{dV} = r_A \quad (E1-2.2)$$

Multiplying both sides of Equation (E1-2.2) by minus one and then substituting Equation (E1-2.1) yields

$$\text{Combine:} \quad -\frac{v_0 dC_A}{dV} = -r_A = kC_A \quad (E1-2.3)$$

Separating the variables and rearranging gives

$$-\frac{v_0}{k} \left(\frac{dC_A}{C_A} \right) = dV$$

Using the conditions at the entrance of the reactor that when $V = 0$, then $C_A = C_{A0}$

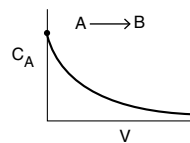
$$-\frac{v_0}{k} \int_{C_{A0}}^{C_A} \frac{dC_A}{C_A} = \int_0^V dV \quad (E1-2.4)$$

Carrying out the integration of Equation (E1-2.4) gives

$$\text{Solve:} \quad \boxed{V = \frac{v_0}{k} \ln \frac{C_{A0}}{C_A}} \quad (E1-2.5)$$

We can also rearrange Equation (E1-2.5) to solve for the concentration of A as a function of reactor volume to obtain

$$C_A = C_{A0} \exp(-kV/v_0)$$



Reactor sizing

Concentration Profile

3. Calculate V . We want to find the volume, V_1 , at which $C_A = \frac{1}{10}C_{A0}$ for $k = 0.23 \text{ min}^{-1}$ and $v_0 = 10 \text{ dm}^3/\text{min}$.

Evaluate:

Substituting C_{A0} , C_A , v_0 , and k in Equation (E1-2.5), we have

$$V = \frac{10 \text{ dm}^3/\text{min}}{0.23 \text{ min}^{-1}} \ln \frac{C_{A0}}{0.1C_{A0}} = \frac{10 \text{ dm}^3}{0.23} \ln 10 = 100 \text{ dm}^3 \text{ (i.e., 100 L; } 0.1 \text{ m}^3\text{)}$$

Let's calculate the volume to reduce the entering concentration to $C_A = 0.01 C_{A0}$. Again using Equation (E1-2.5)

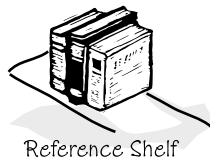
$$V = \frac{10 \text{ dm}^3/\text{min}}{0.23 \text{ min}^{-1}} \ln \frac{C_{A0}}{0.01C_{A0}} = \frac{10 \text{ dm}^3}{0.23} \ln 100 = 200 \text{ dm}^3$$

Note: We see that a larger reactor (200 dm^3) is needed to reduce the exit concentration to a smaller fraction of the entering concentration (e.g., $C_A = 0.01 C_{A0}$).

We see that a reactor volume of 0.1 m^3 is necessary to convert 90% of species A entering into product B for the parameters given.

Analysis: For this irreversible liquid-phase first order reaction (i.e., $-r_A = kC_A$) being carried out in a PFR, the concentration of the reactant decreases exponentially down the length (i.e., volume V) of the reactor. The more species A consumed and converted to product B, the larger must be the reactor volume V . The purpose of the example was to give a vision of the types of calculations we will be carrying out as we study chemical reaction engineering (CRE).

1.5 Industrial Reactors²

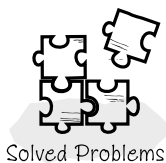


Be sure to view the actual photographs of industrial reactors on the CRE Web site. There are also links to view reactors on different Web sites. The CRE Web site also includes a portion of the *Visual Encyclopedia of Equipment*, encyclopedia.che.engin.umich.edu, "Chemical Reactors" developed by Dr. Susan Montgomery and her students at the University of Michigan. Also see *Professional Reference Shelf* on the CRE Web site for "Reactors for Liquid-Phase and Gas-Phase Reactions," along with photos of industrial reactors, and Expanded Material on the CRE Web site.

In this chapter, and on the CRE Web site, we've introduced each of the major types of industrial reactors: batch, stirred tank, tubular, and fixed bed (packed bed). Many variations and modifications of these commercial reactors (e.g., semibatch, fluidized bed) are in current use; for further elaboration, refer to the detailed discussion of industrial reactors given by Walas.³

² *Chem. Eng.*, 63(10), 211 (1956). See also *AIChE Modular Instruction Series E*, 5 (1984).

³ S. M. Walas, *Reaction Kinetics for Chemical Engineers* (New York: McGraw-Hill, 1959), Chapter 11.



The CRE Web site describes industrial reactors, along with typical feed and operating conditions. In addition, two solved example problems for Chapter 1 can be found on the CRE Web site.

Closure. The goal of this text is to weave the fundamentals of chemical reaction engineering into a structure or algorithm that is easy to use and apply to a variety of problems. We have just finished the first building block of this algorithm: mole balances.

Mole Balance

This algorithm and its corresponding building blocks will be developed and discussed in the following chapters:

- Mole Balance, Chapters 1 and 2
- Rate Law, Chapter 3
- Stoichiometry, Chapter 4
- Combine, Chapter 5
- Evaluate, Chapter 5
- Energy Balance, Chapters 11 through 13

With this algorithm, one can approach and solve chemical reaction engineering problems through logic rather than memorization.

SUMMARY

Each chapter summary gives the key points of the chapter that need to be remembered and carried into succeeding chapters.

1. A mole balance on species j , which enters, leaves, reacts, and accumulates in a system volume V , is

$$F_{j0} - F_j + \int_V r_j dV = \frac{dN_j}{dt} \quad (\text{S1-1})$$

If, and only if, the contents of the reactor are well mixed will the mole balance (Equation (S1-1)) on species A give

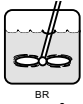
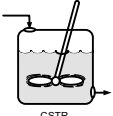

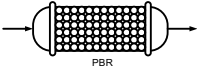
$$F_{A0} - F_A + r_A V = \frac{dN_A}{dt} \quad (\text{S1-2})$$

2. The kinetic rate law for r_j is
 - The rate of formation of species j per unit volume (e.g., mol/s·dm³)
 - Solely a function of the properties of reacting materials and reaction conditions (e.g., concentration [activities], temperature, pressure, catalyst, or solvent [if any]) and does not depend on reactor type
 - An intensive quantity (i.e., it does not depend on the total amount)
 - An algebraic equation, not a differential equation (e.g., $-r_A = kC_A$, $-r_A = kC_A^2$)

For homogeneous catalytic systems, typical units of $-r_j$ may be gram moles per second per liter; for heterogeneous systems, typical units of r'_j may be gram moles per second per gram of catalyst. By convention, $-r_A$ is the rate of disappearance of species A and r_A is the rate of formation of species A.

3. Mole balances on species A in four common reactors are shown in Table S1-1.

TABLE S1-1 SUMMARY OF REACTOR MOLE BALANCES

Reactor	Comment	Mole Balance Differential Form	Algebraic Form	Integral Form
 BR	No spatial variations	$\frac{dN_A}{dt} = r_A V$		$t_1 = \int_{N_{A1}}^{N_{A0}} \frac{dN_A}{-r_A V}$
 CSTR	No spatial variations, steady state	—	$V = \frac{F_{A0} - F_A}{-r_A}$	—
 PFR	Steady state	$\frac{dF_A}{dV} = r_A$		$V_1 = \int_{F_{A1}}^{F_{A0}} \frac{dF_A}{-r_A}$
 PBR	Steady state	$\frac{dF_A}{dW} = r'_A$		$W_1 = \int_{F_{A1}}^{F_{A0}} \frac{dF_A}{-r'_A}$

CRE WEB SITE MATERIALS

- **Expanded Materials**

1. *Industrial Reactors*

- **Learning Resources**

1. *Summary Notes*

2. *Web Material*

- A. Problem-Solving Algorithm

- B. Getting Unstuck on a Problem

This Web site gives tips on how to overcome mental barriers in problem solving.

- C. Smog in L.A. Web module includes a *Living Example Problem*.

B. Getting Unstuck



C. Smog in L.A.



Fotografiert von ©2002 Hank Good.

3. *Interactive Computer Games*
A. Quiz Show I



4. *Solved Problems*

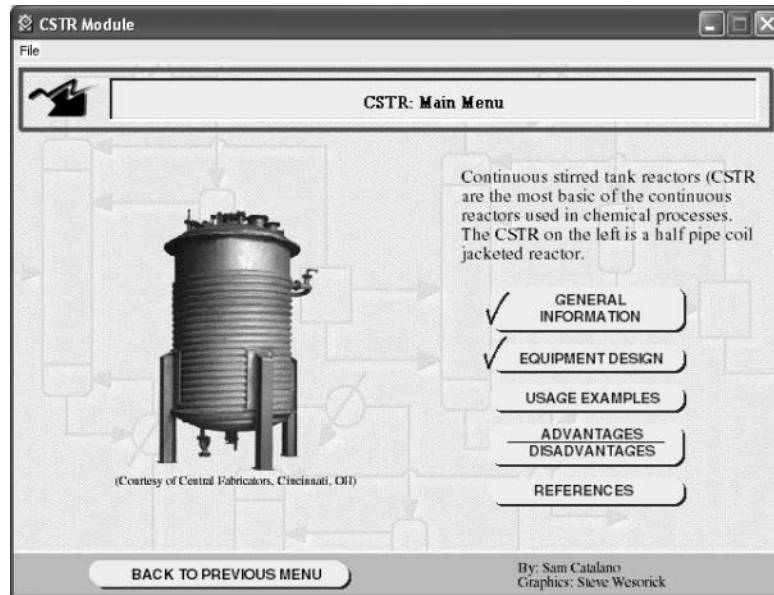
CDP1-A_B Batch Reactor Calculations: A Hint of Things to Come

- **FAQ [Frequently Asked Questions]**—In Updates/FAQ icon section
- **Professional Reference Shelf**
R1.1 *Photos of Real Reactors*



R1.2 *Reactor Section of the Visual Encyclopedia of Equipment (encyclopedia.che.engin.umich.edu)*

This section of the CRE Web site shows industrial equipment and discusses its operation. The reactor portion of this encyclopedia is included on the CRE Web site.



R1.3 *Industrial Reactors*

A. Liquid Phase

- Reactor sizes and costs
- Battery of stirred tanks
- Semibatch

B. Gas Phase

- Costs
- Fluidized bed schematic

R1.4 *Top Ten List of Chemical Products and Chemical Companies*

QUESTIONS AND PROBLEMS

I wish I had an answer for that, because I'm getting tired of answering that question.

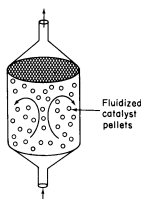
—Yogi Berra, New York Yankees
Sports Illustrated, June 11, 1984

The subscript to each of the problem numbers indicates the level of difficulty, i.e., A, least difficult; B, moderate difficulty; C, fairly difficult; D, (double black diamond), most difficult. A = ● B = ■ C = ◆ D = ◆◆
For example, P1-5_B means "1" is the Chapter number, "5" is the problem number, "B" is the problem difficulty, in this case B means moderate difficulty.

Before solving the problems, state or sketch qualitatively the expected results or trends.

Questions

- Q1-1_A** Read through the Preface. Write a paragraph describing both the content goals and the intellectual goals of the course and text. Also describe what's on the Web site and how the Web site can be used with the text and course.
- Q1-2_A** View the photos and schematics on the Web site under Essentials of Chemical Reaction Engineering—Chapter 1. Look at the QuickTime videos. Write a paragraph describing two or more of the reactors. What similarities and differences do you observe between the reactors on the Web (e.g., *www.loe-bequipment.com*), on the Web site, and in the text? How do the used reactor prices compare with those in Table 1-1?
- Q1-3_A** Surf the Web and the CRE Web site (*www.umich.edu/~elements/5e/index.html*). Go on a scavenger hunt using the summary notes for Chapter 1 on the Web site. Take a quick look at the Web Modules and list the ones that you feel are the most novel applications of CRE.
- Q1-4_A** What does a negative number for the rate of formation of species (e.g., species A), $r_A = -3 \text{ mol/dm}^3 \cdot \text{s}$, signify? What does a positive number signify? Explain.
- Q1-5_A** What assumptions were made in the derivation of the design equation for:
- The batch reactor (BR)?
 - The CSTR?
 - The plug-flow reactor (PFR)?
 - The packed-bed reactor (PBR)?
 - State in words the meanings of $-r_A$ and $-r'_A$.
- Q1-6_A** Use the mole balance to derive an equation analogous to Equation (1-7) for a fluidized CSTR containing catalyst particles in terms of the catalyst weight, W , and other appropriate terms.



$$W = \frac{F_{A0} - F_A}{-r'_A} \quad (\text{Q1-6})$$

Figure Q1-6 Fluidized Bed CSTR.

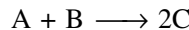
Problems

- P1-1_A** (a) **Revisit Example 1-1.** Rework this example using Equation (3-1) on page 71.
 (b) **Revisit Example 1-2.** Calculate the volume of a CSTR for the conditions used to calculate the plug-flow reactor volume in Example 1-2. Which volume is larger, the PFR or the CSTR? Explain why. Suggest two ways to work this problem incorrectly. [*Ans.*: $V_{\text{CSTR}} = 391 \text{ m}^3$]
 (c) **Revisit Example 1-2.** Calculate the time to reduce the number of moles of A to 1% of its initial value in a constant-volume batch reactor for the same reaction and data in Example 1-2. Suggest two ways to work this problem incorrectly. [*Ans.*: $t = 20 \text{ min}$]
- P1-2_A** (a) Find the Interactive Computer Games (ICG) on the CRE Web site. Play this game and then record your performance number, which indicates your mastery of the material.

ICG Quiz Show

Mole Balance	Reactions	Rate Laws
100	100	100
200	200	200
300	300	300

ICG Kinetics Challenge 1 Performance # _____

P1-3_A The reaction

takes place in an unsteady CSTR. The feed is only A and B in equimolar proportions. Which of the following sets of equations gives the correct set of mole balances on A, B, and C? Species A and B are disappearing and species C is being formed. Circle the correct answer where all the mole balances are correct.

$$\begin{aligned} \text{(a)} \quad F_{B0} - F_A - \int^V r_A dV &= \frac{dN_A}{dt} \\ F_{B0} - F_B - \int^V r_A dV &= \frac{dN_B}{dt} \\ -F_C + 2 \int^V r_A dV &= \frac{dN_C}{dt} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad F_{A0} - F_A + \int^V r_A dV &= \frac{dN_A}{dt} \\ F_{A0} - F_B + \int^V r_A dV &= \frac{dN_B}{dt} \\ -F_C - 2 \int^V r_A dV &= \frac{dN_C}{dt} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad F_{A0} - F_A + \int_0^V r_A dV &= \frac{dN_A}{dt} \\ F_{A0} - F_B + \int^V r_A dV &= \frac{dN_B}{dt} \\ F_C + \int^V r_C dV &= \frac{dN_C}{dt} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad F_{B0} - F_A - \int^V r_A dV &= \frac{dN_A}{dt} \\ F_{B0} - F_{A0} - \int^V r_A dV &= \frac{dN_B}{dt} \\ -F_C + \int^V r_C dV &= \frac{dN_C}{dt} \end{aligned}$$

(e) None of the above.

P1-4_B Schematic diagrams of the Los Angeles basin are shown in Figure P1-4_B. The basin floor covers approximately 700 square miles ($2 \times 10^{10} \text{ ft}^2$) and is almost completely surrounded by mountain ranges. If one assumes an inversion height in the basin of 2,000 ft, the corresponding volume of air in the basin is $4 \times 10^{13} \text{ ft}^3$. We shall use this system volume to model the accumulation and depletion of air pollutants. As a very rough first approximation, we shall treat the Los Angeles basin as a well-mixed container (analogous to a CSTR) in which there are no spatial variations in pollutant concentrations.

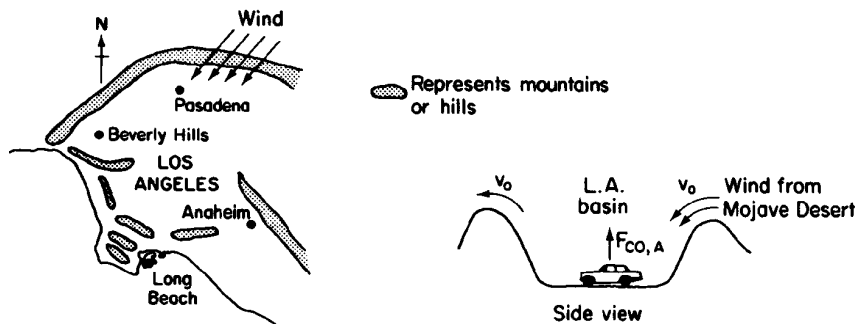


Figure P1-4_B Schematic diagrams of the Los Angeles basin.

We shall perform an unsteady-state mole balance (Equation (1-4)) on CO as it is depleted from the basin area by a Santa Ana wind. Santa Ana winds are high-velocity winds that originate in the Mojave Desert just to the northeast of Los Angeles. Load the **Smog in Los Angeles Basin Web Module**. Use the data in the module to work parts 1–12 (a) through (h) given in the module. Load the **Living Example Polymath code** and explore the problem. For part (i), vary the parameters v_0 , a , and b , and write a paragraph describing what you find.

There is heavier traffic in the L.A. basin in the mornings and in the evenings as workers go to and from work in downtown L.A. Consequently, the flow of CO into the L.A. basin might be better represented by the sine function over a 24-hour period.

P1-5_B The reaction



is to be carried out isothermally in a continuous-flow reactor. The entering volumetric flow rate v_0 is 10 dm³/h. (Note: $F_A = C_A v$. For a constant volumetric flow rate $v = v_0$, then $F_A = C_A v_0$. Also, $C_{A0} = F_{A0}/v_0 = ([5 \text{ mol/h}]/[10 \text{ dm}^3/\text{h}]) 0.5 \text{ mol/dm}^3$.)

Calculate both the CSTR and PFR reactor volumes necessary to consume 99% of A (i.e., $C_A = 0.01C_{A0}$) when the entering molar flow rate is 5 mol/h, assuming the reaction rate $-r_A$ is

(a) $-r_A = k$ with $k = 0.05 \frac{\text{mol}}{\text{h} \cdot \text{dm}^3}$ [Ans.: $V_{\text{CSTR}} = 99 \text{ dm}^3$]

(b) $-r_A = kC_A$ with $k = 0.0001 \text{ s}^{-1}$

(c) $-r_A = kC_A^2$ with $k = 300 \frac{\text{dm}^3}{\text{mol} \cdot \text{h}}$ [Ans.: $V_{\text{CSTR}} = 660 \text{ dm}^3$]

(d) Repeat (a), (b), and/or (c) to calculate the time necessary to consume 99.9% of species A in a 1000 dm³ constant-volume batch reactor with $C_{A0} = 0.5 \text{ mol/dm}^3$.

P1-6_B This problem focuses on using Polymath, an ordinary differential equation (ODE) solver, and also a nonlinear equation (NLE) solver. These equation solvers will be used extensively in later chapters. Information on how to obtain and load the Polymath Software is given in **Appendix D** and on the CRE Web site.

(a) There are initially 500 rabbits (x) and 200 foxes (y) on Farmer Oat's property near Riça, Jofostan. Use Polymath or MATLAB to plot the concentration of foxes and rabbits as a function of time for a period of up to 500 days. The predator-prey relationships are given by the following set of coupled ordinary differential equations:

$$\frac{dx}{dt} = k_1 x - k_2 x \cdot y$$

$$\frac{dy}{dt} = k_3 x \cdot y - k_4 y$$



Constant for growth of rabbits $k_1 = 0.02 \text{ day}^{-1}$

Constant for death of rabbits $k_2 = 0.00004/(\text{day} \times \text{no. of foxes})$

Constant for growth of foxes after eating rabbits $k_3 = 0.0004/(\text{day} \times \text{no. of rabbits})$

Constant for death of foxes $k_4 = 0.04 \text{ day}^{-1}$

What do your results look like for the case of $k_3 = 0.00004/(\text{day} \times \text{no. of rabbits})$ and $t_{\text{final}} = 800$ days? Also, plot the number of foxes versus the number of rabbits. Explain why the curves look the way they do. **Polymath Tutorial** (<https://www.youtube.com/watch?v=nyJmt6cTiL4>)

(b) Use Polymath or MATLAB to solve the following set of nonlinear algebraic equations

$$x^3 y - 4y^2 + 3x = 1$$

$$6y^2 - 9xy = 5$$

with initial guesses of $x = 2$, $y = 2$. Try to become familiar with the edit keys in Polymath and MATLAB. See the CRE Web site for instructions

Screen shots on how to run Polymath are shown at the end of Summary Notes for Chapter 1 or on the CRE Web site, www.umich.edu/~elements/5e/software/polymath-tutorial.html.

P1-7_A **Enrico Fermi (1901–1954) Problems (EFP).** Enrico Fermi was an Italian physicist who received the Nobel Prize for his work on nuclear processes. Fermi was famous for his “Back of the Envelope Order of Magnitude Calculation” to obtain an estimate of the answer through *logic* and then to make reasonable assumptions. He used a process to set bounds on the answer by saying it is probably larger than one number and smaller than another, and arrived at an answer that was within a factor of 10. See <http://mathforum.org/workshops/sum96/interdisc/sheila2.html>.

Enrico Fermi Problem

(a) **EFP #1.** How many piano tuners are there in the city of Chicago? Show the steps in your reasoning.

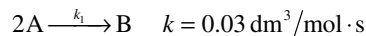
1. Population of Chicago _____
2. Number of people per household _____
3. Etc. _____

An answer is given on the CRE Web site under Summary Notes for Chapter 1.

(b) **EFP #2.** How many square meters of pizza were eaten by an undergraduate student body population of 20,000 during the Fall term 2016?

(c) **EFP #3.** How many bathtubs of water will the average person drink in a lifetime?

P1-8_A **What is wrong with this solution?** The irreversible liquid phase second order reaction ($-r_A = kC_A^2$)



is carried out in a CSTR. The entering concentration of A, C_{A0} , is 2 molar. and the exit concentration of A, C_A is 0.1 molar. The volumetric flow rate, v_0 , is constant at 3 dm³/s. What is the corresponding reactor volume?

Solution

1. Mole Balance

$$V = \frac{F_{A0} - F_A}{-r_A}$$

2. Rate Law (2nd order)

$$-r_A = kC_A^2$$

3. Combine

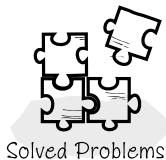
$$V = \frac{F_{A0} - F_A}{kC_A^2}$$

$$4. F_{A0} = v_0 C_{A0} = \frac{3 \text{ dm}^3}{\text{s}} \cdot \frac{2 \text{ molA}}{\text{dm}^3} = \frac{6 \text{ molA}}{\text{s}}$$

$$5. F_A = v_0 C_A = \frac{3 \text{ dm}^3}{\text{s}} \cdot \frac{0.1 \text{ molA}}{\text{dm}^3} = \frac{0.3 \text{ molA}}{\text{s}}$$

$$6. V = \frac{(6 - 0.3) \frac{\text{mol}}{\text{s}}}{\left(0.03 \frac{\text{dm}^3}{\text{mol} \cdot \text{s}}\right) \left(2 \frac{\text{mol}}{\text{dm}^3}\right)^2} = 47.5 \text{ dm}^3$$

For more puzzles on what's wrong with this solution, see additional material for each chapter on the CRE Web site home page, under "Expanded Material."



NOTE TO INSTRUCTORS: Additional problems (cf. those from the preceding editions) can be found in the solutions manual and on the CRE Web site. These problems could be photocopied and used to help reinforce the fundamental principles discussed in this chapter.

SUPPLEMENTARY READING

1. For further elaboration of the development of the general balance equation, see not only the Web site www.umich.edu/~elements/5e/index.html but also

FELDER, R. M., and R. W. ROUSSEAU, *Elementary Principles of Chemical Processes*, 3rd ed. New York: Wiley, 2000, Chapter 4.

SANDERS, R. J., *The Anatomy of Skiing*. Denver, CO: Golden Bell Press, 1976.

2. A detailed explanation of a number of topics in this chapter can be found in the tutorials.

CRYNES, B. L., and H. S. FOGLER, eds., *AIChE Modular Instruction Series E: Kinetics*, Vols. 1 and 2. New York: AIChE, 1981.

3. A discussion of some of the most important industrial processes is presented by

AUSTIN, G. T., *Shreve's Chemical Process Industries*, 5th ed. New York: McGraw-Hill, 1984.

Conversion 2 and Reactor Sizing

Be more concerned with your character than with your reputation, because character is what you really are while reputation is merely what others think you are.

—John Wooden, head coach, UCLA Bruins

Overview. In the first chapter, the general mole balance equation was derived and then applied to the four most common types of industrial reactors. A balance equation was developed for each reactor type and these equations are summarized in Table S1-1 in Chapter 1. In Chapter 2, we will show how to size and arrange these reactors conceptually, so that the reader may see the structure of CRE design and will not get lost in the mathematical details.

In this chapter, we

- Define conversion
- Rewrite all balance equations for the four types of industrial reactors in Chapter 1 in terms of conversion, X
- Show how to size (i.e., determine the reactor volume) these reactors once the relationship between the reaction rate and conversion is known—i.e., given $-r_A = f(X)$
- Show how to compare CSTR and PFR sizes
- Show how to decide the best arrangements for reactors in series, a most important principle

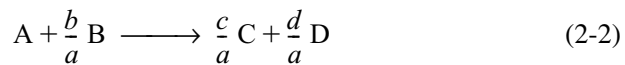
In addition to being able to determine CSTR and PFR sizes given the rate of reaction as a function of conversion, you will be able to calculate the overall conversion and reactor volumes for reactors arranged in series.

2.1 Definition of Conversion

In defining conversion, we choose one of the reactants as the basis of calculation and then relate the other species involved in the reaction to this basis. In virtually all instances we must choose the limiting reactant as the basis of calculation. We develop the stoichiometric relationships and design equations by considering the general reaction



The uppercase letters represent chemical species, and the lowercase letters represent stoichiometric coefficients. We shall choose species A as our limiting reactant and, thus, our *basis of calculation*. The limiting reactant is the reactant that will be completely consumed first after the reactants have been mixed. Next, we divide the reaction expression through by the stoichiometric coefficient of species A , in order to arrange the reaction expression in the form



to put every quantity on a “per mole of A ” basis, our limiting reactant.

Now we ask such questions as “How can we quantify how far a reaction [e.g., Equation (2-2)] proceeds to the right?” or “How many moles of C are formed for every mole of A consumed?” A convenient way to answer these questions is to define a parameter called *conversion*. The conversion X_A is the number of moles of A that have reacted per mole of A fed to the system:

Definition of X

$$X_A = \frac{\text{Moles of A reacted}}{\text{Moles of A fed}}$$

Because we are defining conversion with respect to our basis of calculation [A in Equation (2-2)], we eliminate the subscript A for the sake of brevity and let $X \equiv X_A$. For irreversible reactions, the maximum conversion is 1.0, i.e., complete conversion. For reversible reactions, the maximum conversion is the equilibrium conversion X_e (i.e., $X_{\max} = X_e$). We will take a closer look at equilibrium conversion in Chapter 4.

2.2 Batch Reactor Design Equations

In most batch reactors, the longer a reactant stays in the reactor, the more the reactant is converted to product until either equilibrium is reached or the reactant is exhausted. Consequently, in batch systems the conversion X is a function of the time the reactants spend in the reactor. If N_{A0} is the number of moles of A initially present in the reactor (i.e., $t = 0$), then the total number of moles of A that have reacted (i.e., have been consumed) after a time t is $[N_{A0}X]$.

$$[\text{Moles of A reacted (consumed)}] = [\text{Moles of A fed}] \cdot \left[\frac{\text{Moles of A reacted}}{\text{Moles of A fed}} \right]$$

$$\left[\begin{array}{c} \text{Moles of A} \\ \text{reacted} \\ \text{(consumed)} \end{array} \right] = [N_{A0}] \cdot [X] \quad (2-3)$$

Now, the number of moles of A that remain in the reactor after a time t , N_A , can be expressed in terms of N_{A0} and X :

$$\left[\begin{array}{c} \text{Moles of A} \\ \text{in reactor} \\ \text{at time } t \end{array} \right] = \left[\begin{array}{c} \text{Moles of A} \\ \text{initially fed} \\ \text{to reactor at} \\ t = 0 \end{array} \right] - \left[\begin{array}{c} \text{Moles of A that} \\ \text{have been con-} \\ \text{sumed by chemical} \\ \text{reaction} \end{array} \right]$$

$$[N_A] = [N_{A0}] - [N_{A0}X]$$

The number of moles of A in the reactor after a conversion X has been achieved is

Moles of A in the
reactor at a time t

$$N_A = N_{A0} - N_{A0}X = N_{A0}(1 - X) \quad (2-4)$$

When no spatial variations in reaction rate exist, the mole balance on species A for a batch system is given by the following equation [cf. Equation (1-5)]:

$$\frac{dN_A}{dt} = r_A V \quad (2-5)$$

This equation is valid whether or not the reactor volume is constant. In the general reaction, Equation (2-2), reactant A is disappearing; therefore, we multiply both sides of Equation (2-5) by -1 to obtain the mole balance for the batch reactor in the form

$$-\frac{dN_A}{dt} = (-r_A)V$$

The rate of disappearance of A, $-r_A$, in this reaction might be given by a rate law similar to Equation (1-2), such as $-r_A = kC_A C_B$.

For batch reactors, we are interested in determining how long to leave the reactants in the reactor to achieve a certain conversion X . To determine this length of time, we write the mole balance, Equation (2-5), in terms of conversion by differentiating Equation (2-4) with respect to time, remembering that N_{A0} is the number of moles of A initially present in the reactor and is therefore a constant with respect to time.

$$\frac{dN_A}{dt} = 0 - N_{A0} \frac{dX}{dt}$$

Combining the above with Equation (2-5) yields

$$-N_{A0} \frac{dX}{dt} = r_A V$$

For a batch reactor, the design equation in differential form is

Batch reactor (BR)
design equation

$$N_{A0} \frac{dX}{dt} = -r_A V \quad (2-6)$$

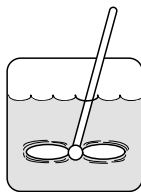
We call Equation (2-6) the differential form of the **design equation** for a batch reactor because we have written the mole balance in terms of conversion. The differential forms of the batch reactor mole balances, Equations (2-5) and (2-6), are often used in the interpretation of reaction rate data (Chapter 7) and for reactors with heat effects (Chapters 11–13), respectively. Batch reactors are frequently used in industry for both gas-phase and liquid-phase reactions. The laboratory bomb calorimeter reactor is widely used for obtaining reaction rate data. Liquid-phase reactions are frequently carried out in batch reactors when small-scale production is desired or operating difficulties rule out the use of continuous-flow systems.

To determine the time to achieve a specified conversion X , we first separate the variables in Equation (2-6) as follows:

$$dt = N_{A0} \frac{dX}{-r_A V}$$

Batch time t
to achieve a
conversion X

This equation is now integrated with the limits that the reaction begins at time equals zero where there is no conversion initially (when $t = 0$, $X = 0$) and ends at time t when a conversion X is achieved (i.e., when $t = t$, then $X = X$). Carrying out the integration, we obtain the time t necessary to achieve a conversion X in a batch reactor



Batch Design
Equation

$$t = N_{A0} \int_0^X \frac{dX}{-r_A V} \quad (2-7)$$

The longer the reactants are left in the reactor, the greater the conversion will be. Equation (2-6) is the differential form of the design equation, and Equation (2-7) is the integral form of the design equation for a batch reactor.

2.3 Design Equations for Flow Reactors

For a batch reactor, we saw that conversion increases with time spent in the reactor. For continuous-flow systems, this time usually increases with increasing reactor volume, e.g., the bigger/longer the reactor, the more time it will take the reactants to flow completely through the reactor and thus, the more time to react. Consequently, the conversion X is a function of reactor volume V . If F_{A0} is the molar flow rate of species A fed to a system operated at steady state, the molar rate at which species A is reacting *within* the entire system will be $F_{A0}X$.

$$[F_{A0}] \cdot [X] = \frac{\text{Moles of A fed}}{\text{time}} \cdot \frac{\text{Moles of A reacted}}{\text{Moles of A fed}}$$

$$[F_{A0} \cdot X] = \frac{\text{Moles of A reacted}}{\text{time}}$$

The molar feed rate of A *to* the system *minus* the rate of reaction of A within the system *equals* the molar flow rate of A leaving the system F_A . The preceding sentence can be expressed mathematically as

$$\left[\begin{array}{c} \text{Molar flow rate} \\ \text{at which A is} \\ \text{fed to the system} \end{array} \right] - \left[\begin{array}{c} \text{Molar rate at} \\ \text{which A is} \\ \text{consumed within} \\ \text{the system} \end{array} \right] = \left[\begin{array}{c} \text{Molar flow rate} \\ \text{at which A leaves} \\ \text{the system} \end{array} \right]$$

$$[F_{A0}] - [F_{A0}X] = [F_A]$$

Rearranging gives

$$\boxed{F_A = F_{A0}(1 - X)} \quad (2-8)$$

The entering molar flow rate of species A, F_{A0} (mol/s), is just the product of the entering concentration, C_{A0} (mol/dm³), and the entering volumetric flow rate, v_0 (dm³/s).

$$\boxed{F_{A0} = C_{A0}v_0} \quad (2-9)$$

Liquid phase *For liquid systems*, the volumetric flow rate, v , is constant and equal to v_0 , and C_{A0} is commonly given in terms of molarity, for example, $C_{A0} = 2$ mol/dm³.

For gas systems, C_{A0} can be calculated from the entering mole fraction, y_{A0} , the temperature, T_0 , and pressure, P_0 , using the ideal gas law or some other gas law. For an ideal gas (see Appendix B):

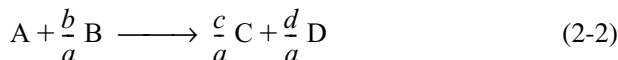
Gas phase

$$C_{A0} = \frac{P_{A0}}{RT_0} = \frac{y_{A0}P_0}{RT_0} \quad (2-10)$$

Now that we have a relationship [Equation (2-8)] between the molar flow rate and conversion, it is possible to express the design equations (i.e., mole balances) in terms of conversion for the *flow* reactors examined in Chapter 1.

2.3.1 CSTR (Also Known as a Backmix Reactor or a Vat)

Recall that the CSTR is modeled as being well mixed such that there are no spatial variations in the reactor. For the general reaction



the CSTR mole balance Equation (1-7) can be arranged to

$$V = \frac{F_{A0} - F_A}{-r_A} \quad (2-11)$$

We now substitute for F_A in terms of F_{A0} and X

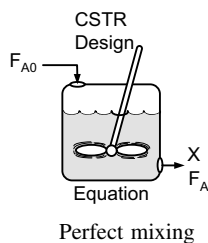
$$F_A = F_{A0} - F_{A0}X \quad (2-12)$$

and then substitute Equation (2-12) into (2-11)

$$V = \frac{F_{A0} - (F_{A0} - F_{A0}X)}{-r_A}$$

Simplifying, we see that the CSTR volume necessary to achieve a specified conversion X is

$$\boxed{V = \frac{F_{A0}X}{(-r_A)_{\text{exit}}} \quad (2-13)$$



Evaluate $-r_A$ at the CSTR exit conditions!!

Because the reactor is *perfectly mixed*, the exit composition from the reactor is identical to the composition inside the reactor, and, therefore, the rate of reaction, $-r_A$, is evaluated at the exit conditions.

2.3.2 Tubular Flow Reactor (PFR)

We model the tubular reactor as having the fluid flowing in plug flow—i.e., no radial gradients in concentration, temperature, or reaction rate.¹ As the reactants enter and flow axially down the reactor, they are consumed and the

¹ This constraint can be removed when we extend our analysis to nonideal (industrial) reactors in Chapters 16 through 18.

conversion increases along the length of the reactor. To develop the PFR design equation, we first multiply both sides of the tubular reactor design equation (1-12) by -1 . We then express the mole balance equation for species A in the reaction as

$$\frac{-dF_A}{dV} = -r_A \quad (2-14)$$

For a flow system, F_A has previously been given in terms of the entering molar flow rate F_{A0} and the conversion X

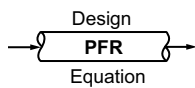
$$F_A = F_{A0} - F_{A0}X \quad (2-12)$$

Differentiating

$$dF_A = -F_{A0}dX$$

and substituting into (2-14) gives the differential form of the design equation for a plug-flow reactor (PFR)

$$\boxed{F_{A0} \frac{dX}{dV} = -r_A} \quad (2-15)$$



We now separate the variables and integrate with the limits $V = 0$ when $X = 0$ to obtain the plug-flow reactor volume necessary to achieve a specified conversion X

$$\boxed{V = F_{A0} \int_0^X \frac{dX}{-r_A}} \quad (2-16)$$

To carry out the integrations in the batch and plug-flow reactor design equations (2-7) and (2-16), as well as to evaluate the CSTR design equation (2-13), we need to know how the reaction rate $-r_A$ varies with the concentration (hence conversion) of the reacting species. This relationship between reaction rate and concentration is developed in Chapter 3.

2.3.3 Packed-Bed Reactor (PBR)

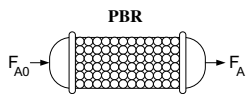
Packed-bed reactors are tubular reactors filled with catalyst particles. In PBRs it is the weight of catalyst W that is important, rather than the reactor volume. The derivation of the differential and integral forms of the design equations for packed-bed reactors are analogous to those for a PFR [cf. Equations (2-15) and (2-16)]. That is, substituting Equation (2-12) for F_A in Equation (1-15) gives

PBR design
equation

$$\boxed{F_{A0} \frac{dX}{dW} = -r'_A} \quad (2-17)$$

The differential form of the design equation [i.e., Equation (2-17)] **must** be used when analyzing reactors that have a pressure drop along the length of the reactor. We discuss pressure drop in packed-bed reactors in Chapter 5.

In the *absence* of pressure drop, i.e., $\Delta P = 0$, we can integrate (2-17) with limits $X = 0$ at $W = 0$, and when $W = W$ then $X = X$ to obtain



$$W = F_{A0} \int_0^X \frac{dX}{-r'_A} \quad (2-18)$$

Equation (2-18) can be used to determine the catalyst weight W (i.e., mass) necessary to achieve a conversion X when the total pressure remains constant.

2.4 Sizing Continuous-Flow Reactors

In this section, we are going to show how we can size CSTRs and PFRs (i.e., determine their reactor volumes) from knowledge of the rate of reaction, $-r_A$, as a function of conversion, X [i.e., $-r_A = f(X)$]. The rate of disappearance of A, $-r_A$, is almost always a function of the concentrations of the various species present (see Chapter 3). When only one reaction is occurring, each of the concentrations can be expressed as a function of the conversion X (see Chapter 4); consequently, $-r_A$ can be expressed as a function of X .

A particularly simple functional dependence, yet one that occurs often, is the first-order dependence

$$-r_A = kC_A = kC_{A0}(1 - X)$$

Here, k is the specific reaction rate and is a function only of temperature, and C_{A0} is the entering concentration of A. We note in Equations (2-13) and (2-16) that the reactor volume is a function of the reciprocal of $-r_A$. For this first-order dependence, a plot of the reciprocal rate of reaction ($1/-r_A$) as a function of conversion yields a curve similar to the one shown in Figure 2-1, where

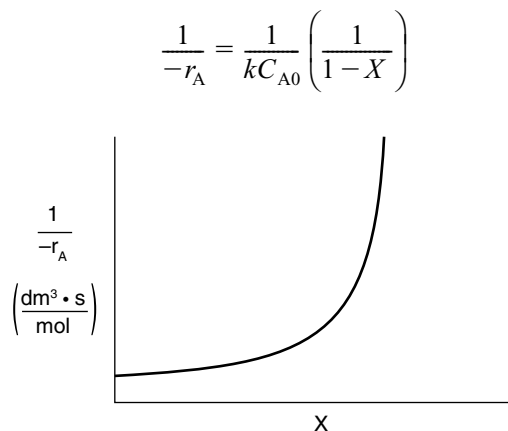


Figure 2-1 Typical curve of the reciprocal rate as a function of conversion.

We can use Figure 2-1 to size CSTRs and PFRs for different entering flow rates. By *sizing* we mean either determine the reactor volume for a specified conversion or determine the conversion for a specified reactor volume. Before sizing flow reactors, let's consider some insights. If a reaction is carried out isothermally, the rate is usually greatest at the start of the reaction when the concentration of reactant is greatest (i.e., when there is negligible conversion [$X \cong 0$]). Hence, the reciprocal rate ($1/-r_A$) will be small. Near the end of the reaction, when the reactant has been mostly used up and thus the concentration of A is small (i.e., the conversion is large), the reaction rate will be small. Consequently, the reciprocal rate ($1/-r_A$) is large.

For all irreversible reactions of greater than zero order (see Chapter 3 for zero-order reactions), as we approach complete conversion where all the limiting reactant is used up, i.e., $X = 1$, the reciprocal rate approaches infinity as does the reactor volume, i.e.

$$A \rightarrow B + C \quad \text{As } X \rightarrow 1, -r_A \rightarrow 0, \text{ thus, } \frac{1}{-r_A} \rightarrow \infty \text{ and therefore } V \rightarrow \infty$$

“To infinity
and beyond”
—Buzz Lightyear

Consequently, we see that an infinite reactor volume is necessary to reach complete conversion, $X = 1.0$.

For reversible reactions (e.g., $A \rightleftharpoons B$), the maximum conversion is the equilibrium conversion X_e . At equilibrium, the reaction rate is zero ($r_A \cong 0$). Therefore,

$$A \rightleftharpoons B + C \quad \text{as } X \rightarrow X_e, -r_A \rightarrow 0, \text{ thus, } \frac{1}{-r_A} \rightarrow \infty \text{ and therefore } V \rightarrow \infty$$

and we see that an infinite reactor volume would also be necessary to obtain the exact equilibrium conversion, $X = X_e$. We will discuss X_e further in Chapter 4.

Examples of Reactor Design and Staging Given $-r_A = f(X)$

To illustrate the design of continuous-flow reactors (i.e., CSTRs and PFRs), we consider the isothermal gas-phase isomerization



We are going to the laboratory to determine the rate of chemical reaction as a function of the conversion of reactant A. The laboratory measurements given in Table 2-1 show the chemical reaction rate as a function of conversion. The temperature was 500 K (440°F), the total pressure was 830 kPa (8.2 atm), and the initial charge to the reactor was pure A. The entering molar flow of A rate is $F_{A0} = 0.4$ mol/s.

If we know $-r_A$ as a function of X , we can size any isothermal reaction system.

TABLE 2-1 RAW DATA[†]

X	0	0.1	0.2	0.4	0.6	0.7	0.8
$-r_A$ (mol/m ³ · s)	0.45	0.37	0.30	0.195	0.113	0.079	0.05

[†] Proprietary coded data courtesy of Jofostan Central Research Laboratory, Çölow, Jofostan, and published in *Jofostan Journal of Chemical Engineering Research*, Volume 21, page 73 (1993).

Recalling the CSTR and PFR design equations, (2-13) and (2-16), we see that the reactor volume varies directly with the molar flow rate F_{A0} and with the reciprocal of $-r_A$, $\left(\frac{1}{-r_A}\right)$, e.g., $V = \left(\frac{F_{A0}}{-r_A}\right)X$. Consequently, to size reactors, we first convert the raw data in Table 2-1, which gives $-r_A$ as a function of X first to $\left(\frac{1}{-r_A}\right)$ as a function of X . Next, we multiply by the entering molar flow rate, F_{A0} , to obtain $\left(\frac{F_{A0}}{-r_A}\right)$ as a function of X as shown in Table 2-2 of the processed data for $F_{A0} = 0.4$ mol/s.

TABLE 2-2 PROCESSED DATA

X	0.0	0.1	0.2	0.4	0.6	0.7	0.8
$-r_A \left(\frac{\text{mol}}{\text{m}^3 \cdot \text{s}}\right)$	0.45	0.37	0.30	0.195	0.113	0.079	0.05
$(1/-r_A) \left(\frac{\text{m}^3 \cdot \text{s}}{\text{mol}}\right)$	2.22	2.70	3.33	5.13	8.85	12.7	20
$(F_{A0}/-r_A)(\text{m}^3)$	0.89	1.08	1.33	2.05	3.54	5.06	8.0

We shall use the data in this table for the next five Example Problems.

To size reactors for different entering molar flow rates, F_{A0} , we would use rows 1 and 3 in Table 2-2 to construct the following figure:

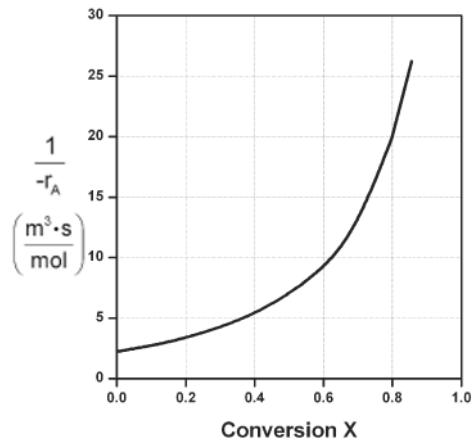


Figure 2-2A Processed data 1.

However, for a given F_{A0} , rather than use Figure 2-2A to size reactors, it is often more advantageous to plot $\left(\frac{F_{A0}}{-r_A}\right)$ as a function of X , which is called a Levenspiel plot. We are now going to carry out a number of examples where we have specified the flow rate F_{A0} at 0.4 mol A/s.

Plotting $\left(\frac{F_{A0}}{-r_A}\right)$ as a function of X using the data in Table 2-2 we obtain the plot shown in Figure 2-2B.

Levenspiel plot

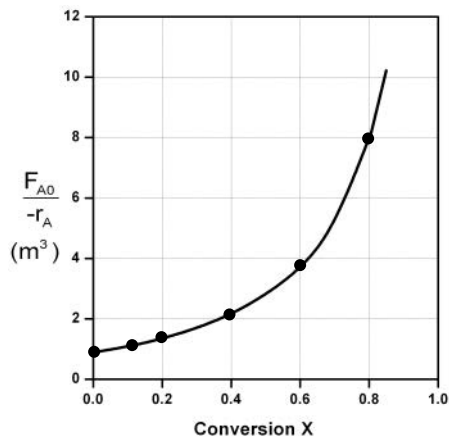


Figure 2-2B Levenspiel plot of processed data 2.

We are now going to use the Levenspiel plot of the processed data (Figure 2-2B) to size a CSTR and a PFR.

Example 2-1 Sizing a CSTR

The reaction described by the data in Table 2-2



is to be carried out in a CSTR. Species A enters the reactor at a molar flow rate of $F_{A0} = 0.4 \frac{\text{mol}}{\text{s}}$, which is the flow rate used to construct Figure 2-2B.

- (a) Using the data in either Table 2-2 or Figure 2-2B, calculate the volume necessary to achieve 80% conversion in a CSTR.
 (b) Shade the area in Figure 2-2B that would give the CSTR volume necessary to achieve 80% conversion.

Solutions

- (a) Equation (2-13) gives the volume of a CSTR as a function of F_{A0} , X , and $-r_A$

$$V = \frac{F_{A0}X}{(-r_A)_{\text{exit}}} \quad (2-13)$$

In a CSTR, the composition, temperature, and conversion of the effluent stream are identical to that of the fluid within the reactor, because perfect mixing is assumed. Therefore, we need to find the value of $-r_A$ (or reciprocal thereof) at $X = 0.8$. From either Table 2-2 or Figure 2-2A, we see that when $X = 0.8$, then

$$\left(\frac{1}{-r_A}\right)_{X=0.8} = 20 \frac{\text{m}^3 \cdot \text{s}}{\text{mol}}$$

Substitution into Equation (2-13) for an entering molar flow rate, F_{A0} , of 0.4 mol A/s and $X = 0.8$ gives

$$V = 0.4 \frac{\text{mol}}{\text{s}} \left(\frac{20 \text{ m}^3 \cdot \text{s}}{\text{mol}} \right) (0.8) = 6.4 \text{ m}^3 \quad (\text{E2-1.1})$$

$$V = 6.4 \text{ m}^3 = 6400 \text{ dm}^3 = 6400 \text{ liters}$$

- (b) Shade the area in Figure 2-2B that yields the CSTR volume. Rearranging Equation (2-13) gives

$$V = \left[\frac{F_{A0}}{-r_A} \right] X \quad (2-13)$$

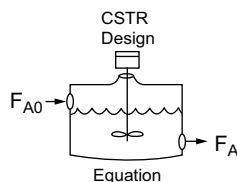
In Figure E2-1.1, the volume is equal to the area of a rectangle with a height ($F_{A0}/-r_A = 8 \text{ m}^3$) and a base ($X = 0.8$). This rectangle is shaded in the figure.

$$V = \left[\frac{F_{A0}}{-r_A} \right]_{X=0.8} (0.8) \quad (\text{E2-1.2})$$

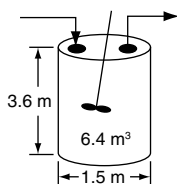
$V =$ Levenspiel rectangle area = height \times width

$$V = [8 \text{ m}^3][0.8] = 6.4 \text{ m}^3 = 6400 \text{ dm}^3 = 6400 \text{ L}$$

The CSTR volume necessary to achieve 80% conversion is 6.4 m³ when operated at 500 K, 830 kPa (8.2 atm), and with an entering molar flow rate of A of 0.4 mol/s. This volume corresponds to a reactor about 1.5 m in diameter and 3.6 m



Representative Industrial CSTR Dimensions



high. It's a large CSTR, but this is a gas-phase reaction, and CSTRs are normally not used for gas-phase reactions. CSTRs are used primarily for liquid-phase reactions.

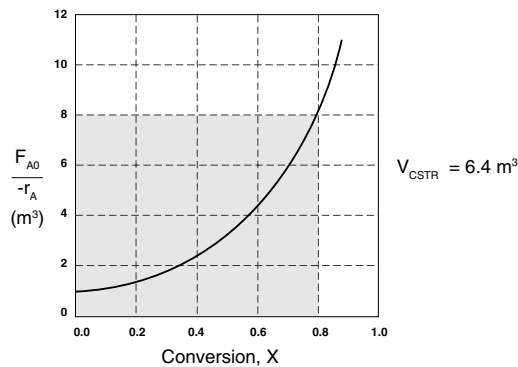


Figure E2-1.1 Levenspiel CSTR plot.

Plots of $(F_{A0}/-r_A)$ vs. X are sometimes referred to as Levenspiel plots (after Octave Levenspiel).

Analysis: Given the conversion, the rate of reaction as a function of conversion along with the molar flow of the species A, we saw how to calculate the volume of a CSTR. From the data and information given, we calculated the volume to be 6.4 m^3 for 80% conversion. We showed how to carry out this calculation using the design equation (2-13) and also using a Levenspiel plot.

Example 2-2 Sizing a PFR

The reaction described by the data in Tables 2-1 and 2-2 is to be carried out in a PFR. The entering molar flow rate of A is again 0.4 mol/s .

- First, use one of the integration formulas given in Appendix A.4 to determine the PFR reactor volume necessary to achieve 80% conversion.
- Next, shade the area in Figure 2-2B that would give the PFR volume necessary to achieve 80% conversion.
- Finally, make a qualitative sketch of the conversion, X , and the rate of reaction, $-r_A$, down the length (volume) of the reactor.

Solution

We start by repeating rows 1 and 4 of Table 2-2 to produce the results shown in Table 2-3.

TABLE 2-3 PROCESSED DATA 2

X	0.0	0.1	0.2	0.4	0.6	0.7	0.8
$(F_{A0}/-r_A)(\text{m}^3)$	0.89	1.08	1.33	2.05	3.54	5.06	8.0

- Numerically evaluate PFR volume.** For the PFR, the differential form of the mole balance is

$$F_{A0} \frac{dX}{dV} = -r_A \quad (2-15)$$

Rearranging and integrating gives

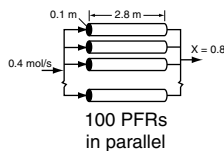
$$V = F_{A0} \int_0^{0.8} \frac{dX}{-r_A} = \int_0^{0.8} \frac{F_{A0}}{-r_A} dX \quad (2-16)$$

We shall use the *five-point quadrature* formula [Equation (A-23)] given in Appendix A.4 to numerically evaluate Equation (2-16). The five-point formula with a final conversion of 0.8 gives four equal segments between $X = 0$ and $X = 0.8$, with a segment length of $\Delta X = \frac{0.8}{4} = 0.2$. The function inside the integral is evaluated at $X = 0$, $X = 0.2$, $X = 0.4$, $X = 0.6$, and $X = 0.8$.

$$V = \frac{\Delta X}{3} \left[\frac{F_{A0}}{-r_A(X=0)} + \frac{4F_{A0}}{-r_A(X=0.2)} + \frac{2F_{A0}}{-r_A(X=0.4)} + \frac{4F_{A0}}{-r_A(X=0.6)} + \frac{F_{A0}}{-r_A(X=0.8)} \right] \quad (E2-2.1)$$

Using values of $[F_{A0}/(-r_A)]$ corresponding to the different conversions in Table 2-3 yields

$$V = \left(\frac{0.2}{3}\right)[0.89 + 4(1.33) + 2(2.05) + 4(3.54) + 8.0]m^3 = \left(\frac{0.2}{3}\right)(32.47 m^3)$$



$$V = 2.165 m^3 = 2165 dm^3$$

The PFR reactor volume necessary to achieve 80% conversion is 2165 dm³. This volume could result from a bank of 100 PFRs that are each 0.1 m in diameter with a length of 2.8 m (e.g., see margin figure or Figures 1-8(a) and (b)).

(b) The PFR volume, i.e., the integral in Equation (2-16), can also be evaluated from the area under the curve of a plot of $(F_{A0}/-r_A)$ versus X .

$$V = \int_0^{0.8} \frac{F_{A0}}{-r_A} dX = \text{Area under the curve between } X = 0 \text{ and } X = 0.8 \quad (\text{see shaded area in Figure E2-2.1})$$

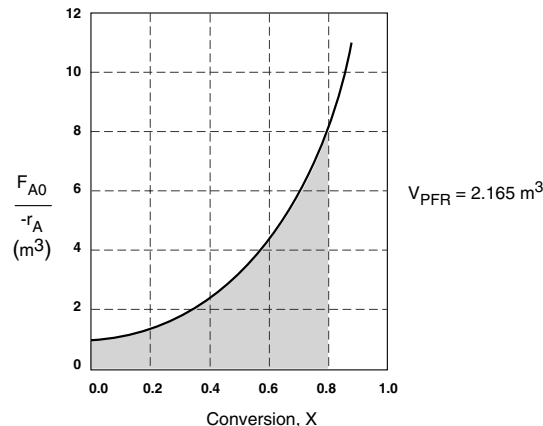


Figure E2-2.1 Levenspiel PFR plot.

PFR

The area under the curve will give the tubular reactor volume necessary to achieve the specified conversion of A. For 80% conversion, the shaded area is roughly equal to 2165 dm³ (2.165 m³).

(c) Sketch the profiles of $-r_A$ and X down the length of the reactor.

We know that as we proceed down the reactor, the conversion increases as more and more reactant is converted to product. Consequently, as the reactant is consumed, the concentration of reactant decreases, as does the rate of disappearance of A for isothermal reactions.

(i) For $X = 0.2$, we calculate the corresponding reactor volume using Simpson's rule [given in Appendix A.4 as Equation (A-21)] with increment $\Delta X = 0.1$ and the data in rows 1 and 4 in Table 2-2.

$$\begin{aligned}
 V &= F_{A0} \int_0^{0.2} \frac{dX}{-r_A} = \frac{\Delta X}{3} \left[\frac{F_{A0}}{-r_A(X=0)} + \frac{4F_{A0}}{-r_A(X=0.1)} + \frac{F_{A0}}{-r_A(X=0.2)} \right] \quad (\text{E2-2.2}) \\
 &= \left[\frac{0.1}{3} [0.89 + 4(1.08) + 1.33] \right] \text{m}^3 = \frac{0.1}{3} (6.54 \text{ m}^3) = 0.218 \text{ m}^3 = 218 \text{ dm}^3 \\
 &= 218 \text{ dm}^3
 \end{aligned}$$

This volume (218 dm³) is the volume at which $X = 0.2$. From Table 2-3, we see the corresponding rate of reaction at $X = 0.2$ is $-r_A = 0.3 \frac{\text{mol}}{\text{dm}^3 \cdot \text{s}}$.

Therefore at $X = 0.2$, then $-r_A = 0.3 \frac{\text{mol}}{\text{dm}^3 \cdot \text{s}}$ and $V = 218 \text{ dm}^3$.

(ii) For $X = 0.4$, we can again use Table 2-3 and Simpson's rule with $\Delta X = 0.2$ to find the reactor volume necessary for a conversion of 40%.

$$\begin{aligned}
 V &= \frac{\Delta X}{3} \left[\frac{F_{A0}}{-r_A(X=0)} + \frac{4F_{A0}}{-r_A(X=0.2)} + \frac{F_{A0}}{-r_A(X=0.4)} \right] \\
 &= \left[\frac{0.2}{3} [0.89 + 4(1.33) + 2.05] \right] \text{m}^3 = 0.551 \text{ m}^3 \\
 &= 551 \text{ dm}^3
 \end{aligned}$$

From Table 2-3 we see that at $X = 0.4$, $-r_A = 0.195 \frac{\text{mol}}{\text{dm}^3 \cdot \text{s}}$ and $V = 551 \text{ dm}^3$.

We can continue in this manner to arrive at Table E2-2.1.

TABLE E2-2.1 CONVERSION AND REACTION RATE PROFILES

X	0	0.2	0.4	0.6	0.8
$-r_A \left(\frac{\text{mol}}{\text{m}^3 \cdot \text{s}} \right)$	0.45	0.30	0.195	0.113	0.05
$V \text{ (dm}^3\text{)}$	0	218	551	1093	2165

The data in Table E2-2.1 are plotted in Figures E2-2.2(a) and (b).

For isothermal reactions, the conversion increases and the rate decreases as we move down the PFR.

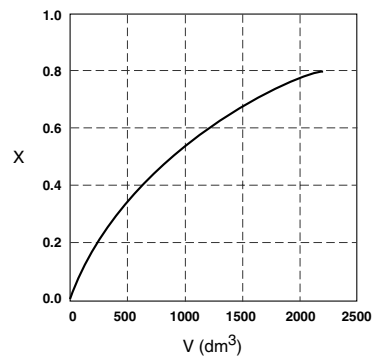


Figure E2-2.2(a) Conversion profile.

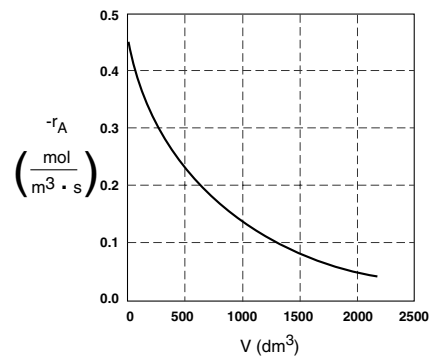


Figure E2-2.2(b) Reaction rate profile.

Analysis: One observes that the reaction rate, $-r_A$, decreases as we move down the reactor while the conversion increases. These plots are typical for reactors operated isothermally.

Example 2-3 Comparing CSTR and PFR Sizes

Compare the volumes of a CSTR and a PFR required for the same conversion using the data in Figure 2-2B. Which reactor would require the smaller volume to achieve a conversion of 80%: a CSTR or a PFR? The entering molar flow rate and the feed conditions are the same in both cases.

Solution

We will again use the data in Table 2-3.

TABLE 2-3 PROCESSED DATA 2

X	0.0	0.1	0.2	0.4	0.6	0.7	0.8
$(F_{A0}/-r_A)(\text{m}^3)$	0.89	1.08	1.33	2.05	3.54	5.06	8.0

The CSTR volume was 6.4 m^3 and the PFR volume was 2.165 m^3 . When we combine Figures E2-1.1 and E2-2.1 on the same graph, Figure 2-3.1(a), we see that the crosshatched area above the curve is the difference in the CSTR and PFR reactor volumes.

For isothermal reactions *greater* than zero order (see Chapter 3), the CSTR volume will always be greater than the PFR volume for the same conversion and reaction conditions (temperature, flow rate, etc.).

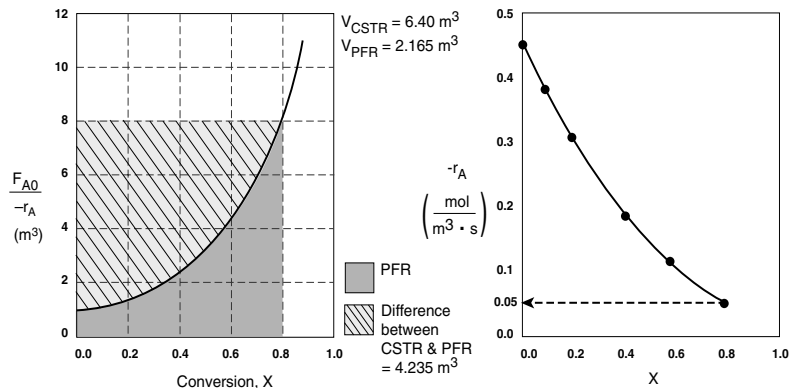


Figure E2-3.1(a) Comparison of CSTR and PFR reactor sizes for $X = 0.8$.

Figure E2-3.1(b) $-r_A$ as a function of X obtained from Table 2-2.1.

Analysis: We see that the reason the isothermal CSTR volume is usually greater than the PFR volume is that the CSTR is always operating at the lowest reaction rate (e.g., $-r_A = 0.05 \text{ mol/m}^3 \cdot \text{s}$ in Figure E2-3.1(b)). The PFR, on the other hand, starts at a high rate at the entrance and gradually decreases to the exit rate, thereby requiring less volume because the volume is inversely proportional to the rate. However, there are exceptions such as autocatalytic reactions, product-inhibited reactions, and nonisothermal exothermic reactions; these trends will not always be the case, as we will see in Chapters 9 and 11.

2.5 Reactors in Series

Many times, reactors are connected in series so that the exit stream of one reactor is the feed stream for another reactor. When this arrangement is used, it is often possible to speed calculations by defining conversion in terms of location at a point downstream rather than with respect to any single reactor. That is, the conversion X is the *total number of moles* of A that have reacted up to that point per mole of A fed to the *first* reactor.

Only valid for
NO side streams!!

For reactors in series

$$X_i = \frac{\text{Total moles of A reacted up to point } i}{\text{Moles of A fed to the first reactor}}$$



However, this definition can *only* be used when the feed stream only enters the first reactor in the series and there are *no* side streams either fed or withdrawn. The molar flow rate of A at point i is equal to the moles of A fed to the first reactor, minus all the moles of A reacted up to point i .

$$F_{A_i} = F_{A0} - F_{A0}X_i$$

For the reactors shown in Figure 2-3, X_1 at point $i = 1$ is the conversion achieved in the PFR, X_2 at point $i = 2$ is the total conversion achieved at this point in the PFR and the CSTR, and X_3 is the total conversion achieved by all three reactors.

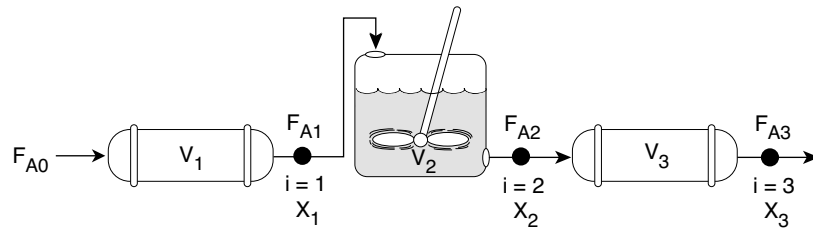


Figure 2-3 Reactors in series.

To demonstrate these ideas, let us consider three different schemes of reactors in series: two CSTRs, two PFRs, and then a combination of PFRs and CSTRs in series. To size these reactors, we shall use laboratory data that give the reaction rate at different conversions.

2.5.1 CSTRs in Series

The first scheme to be considered is the two CSTRs in series shown in Figure 2-4.

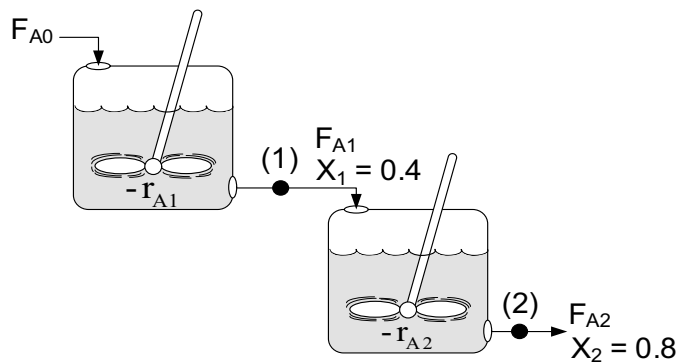


Figure 2-4 Two CSTRs in series.

For the first reactor, the rate of disappearance of A is $-r_{A1}$ at conversion X_1 . A mole balance on reactor 1 gives

$$\mathbf{In - Out + Generation = 0}$$

$$\mathbf{Reactor 1:} \quad F_{A0} - F_{A1} + r_{A1}V_1 = 0 \quad (2-19)$$

The molar flow rate of A at point 1 is

$$F_{A1} = F_{A0} - F_{A0}X_1 \quad (2-20)$$

Combining Equations (2-19) and (2-20), or rearranging

$$\text{Reactor 1} \quad \boxed{V_1 = \frac{F_{A0}X_1}{-r_{A1}}} \quad (2-21)$$

In the second reactor, the rate of disappearance of A, $-r_{A2}$, is evaluated at the conversion of the exit stream of reactor 2, X_2 . A steady-state mole balance on the second reactor is

$$\mathbf{In - Out + Generation = 0}$$

$$\mathbf{Reactor 2:} \quad F_{A1} - F_{A2} + r_{A2}V_2 = 0 \quad (2-22)$$

The molar flow rate of A at point 2 is

$$F_{A2} = F_{A0} - F_{A0}X_2 \quad (2-23)$$

Combining and rearranging

$$V_2 = \frac{F_{A1} - F_{A2}}{-r_{A2}} = \frac{(F_{A0} - F_{A0}X_1) - (F_{A0} - F_{A0}X_2)}{-r_{A2}}$$

$$\text{Reactor 2} \quad \boxed{V_2 = \frac{F_{A0}}{-r_{A2}}(X_2 - X_1)} \quad (2-24)$$

For the second CSTR, recall that $-r_{A2}$ is evaluated at X_2 and then use $(X_2 - X_1)$ to calculate V_2 .

In the examples that follow, we shall again use the molar flow rate of A used in Example 2-1 (i.e., $F_{A0} = 0.4$ mol A/s) and the reaction conditions given in Table 2-3.

Example 2-4 Comparing Volumes for CSTRs in Series

For the two CSTRs in series, 40% conversion is achieved in the first reactor. What is the volume of each of the two reactors necessary to achieve 80% overall conversion of the entering species A? (See Table 2-3.)

TABLE 2-3 PROCESSED DATA 2

X	0.0	0.1	0.2	0.4	0.6	0.7	0.8
$(F_{A0}/-r_A)(\text{m}^3)$	0.89	1.09	1.33	2.05	3.54	5.06	8.0

Solution

For Reactor 1, we observe from either Table 2-3 or Figure 2-2B that when $X = 0.4$, then

$$\left(\frac{F_{A0}}{-r_{A1}}\right)_{X=0.4} = 2.05 \text{ m}^3$$

Then, using Equation (2-13)

$$V_1 = \left(\frac{F_{A0}}{-r_{A1}}\right)_{X_1} X_1 = \left(\frac{F_{A0}}{-r_{A1}}\right)_{0.4} X_1 = (2.05)(0.4) = 0.82 \text{ m}^3 = 820 \text{ dm}^3$$

For Reactor 2, when $X_2 = 0.8$, then $\left(\frac{F_{A0}}{-r_{A2}}\right)_{X=0.8} = 8.0 \text{ m}^3$

using Equation (2-24)

$$V_2 = \left(\frac{F_{A0}}{-r_{A2}}\right)(X_2 - X_1) \quad (2-24)$$

$$V_2 = (8.0 \text{ m}^3)(0.8 - 0.4) = 3.2 \text{ m}^3 = 3200 \text{ dm}^3$$

$$V_2 = 3200 \text{ dm}^3 \text{ (liters)}$$

To achieve the same overall conversion, the total volume for two CSTRs in series is less than that required for one CSTR.

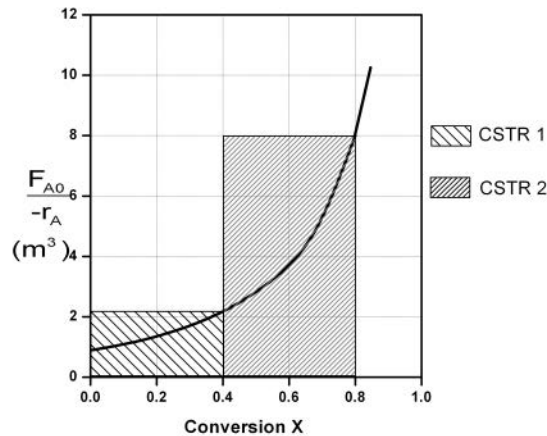


Figure E2-4.1 Two CSTRs in series.

The shaded areas in Figure E2-4.1 can also be used to determine volumes of CSTR 1 and CSTR 2.

Note again that for CSTRs in series, the rate $-r_{A1}$ is evaluated at a conversion of 0.4 and rate $-r_{A2}$ is evaluated at a conversion of 0.8. The total volume for these two reactors in series is

$$V = V_1 + V_2 = 0.82 \text{ m}^3 + 3.2 \text{ m}^3 = 4.02 \text{ m}^3 = 4020 \text{ dm}^3$$

We need only $-r_A = f(X)$ and F_{A0} to size reactors.

By comparison, the volume necessary to achieve 80% conversion in **one** CSTR is

$$V = \left(\frac{F_{A0}}{-r_{A1}} \right) X = (8.0)(0.8) = 6.4 \text{ m}^3 = 6400 \text{ dm}^3$$

Notice in Example 2-5 that the sum of the two CSTR reactor volumes (4.02 m^3) in series is less than the volume of one CSTR (6.4 m^3) to achieve the same overall conversion.

Analysis: When we have reactors in series, we can speed our analysis and calculations by defining an overall conversion at a point in the series, rather than the conversion of each individual reactor. In this example, we saw that 40% was achieved at point 1, the exit to the first reactor, and that a total of 80% conversion was achieved by the time we exit the second reactor.

Approximating a PFR by a Large Number of CSTRs in Series

Consider approximating a PFR with a number of small, equal-volume CSTRs of V_i in series (Figure 2-5). We want to compare the *total volume* of all the CSTRs with the volume of one plug-flow reactor for the same conversion, say 80%.

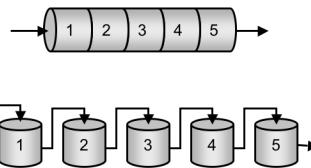
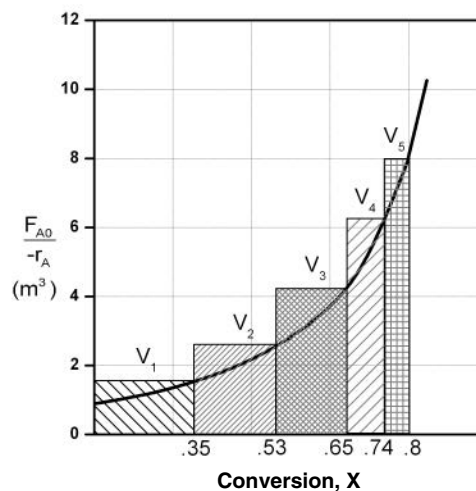


Figure 2-5 Modeling a PFR with CSTRs in series.

From Figure 2-6, we note a very important observation! The total volume to achieve 80% conversion for five CSTRs of equal volume in series is



The fact that we can model a PFR with a large number of CSTRs is an important result.

Figure 2-6 Levenspiel plot showing comparison of CSTRs in series with one PFR.

“roughly” the same as the volume of a PFR. As we make the volume of each CSTR smaller and increase the number of CSTRs, the total volume of the CSTRs in series and the volume of the PFR will become identical. *That is, we can model a PFR with a large number of CSTRs in series.* This concept of using many CSTRs in series to model a PFR will be used later in a number of situations, such as modeling catalyst decay in packed-bed reactors or transient heat effects in PFRs.

2.5.2 PFRs in Series

We saw that two CSTRs in series gave a smaller total volume than a single CSTR to achieve the same conversion. This case does not hold true for the two plug-flow reactors connected in series shown in Figure 2-7.

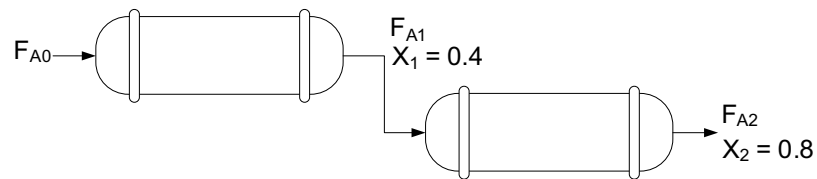


Figure 2-7 Two PFRs in series.

PFRs in series We can see from Figure 2-8 and from the following equation

$$\int_0^{X_2} F_{A0} \frac{dX}{-r_A} \equiv \int_0^{X_1} F_{A0} \frac{dX}{-r_A} + \int_{X_1}^{X_2} F_{A0} \frac{dX}{-r_A}$$

that it is immaterial whether you place two plug-flow reactors in series or have one continuous plug-flow reactor; the total reactor volume required to achieve the same conversion is identical!

The overall conversion of two PFRs in series is the same as one PFR with the same total volume.

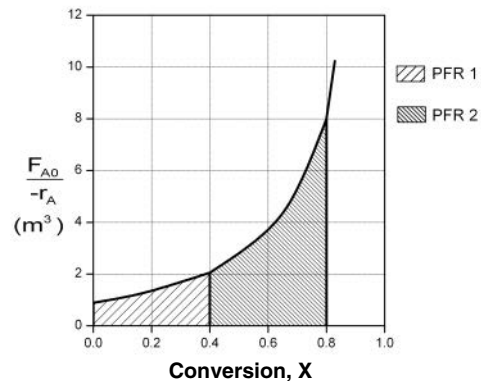
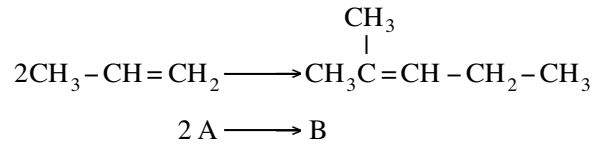


Figure 2-8 Levenspiel plot for two PFRs in series.

2.5.3 Combinations of CSTRs and PFRs in Series

The final sequences we shall consider are combinations of CSTRs and PFRs in series. An industrial example of reactors in series is shown in the photo in Figure 2-9. This sequence is used to dimerize propylene (A) into olefins (B), e.g.,



Not sure if the size of these CSTRs is in the Guinness Book of World Records

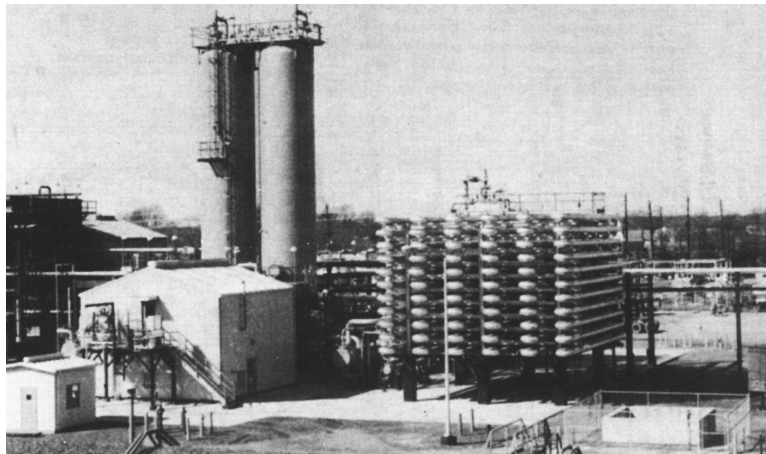


Figure 2-9 Dimersol G (an organometallic catalyst) unit (two CSTRs and one tubular reactor in series) to dimerize propylene into isohexanes. Institut Français du Pétrole process. Photo courtesy of Editions Technip (Institut Français du Pétrole).

A schematic of the industrial reactor system in Figure 2-9 is shown in Figure 2-10.

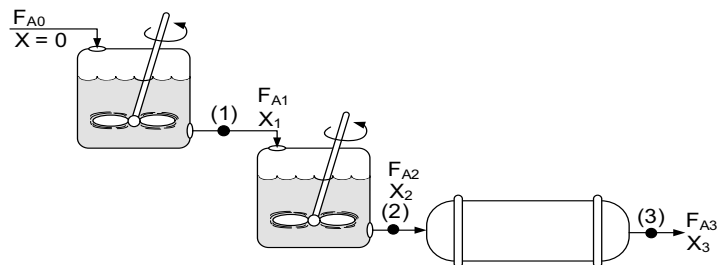


Figure 2-10 Schematic of a real system.

For the sake of illustration, let's assume that the reaction carried out in the reactors in Figure 2-10 follows the same $\left(\frac{F_{A0}}{-r_A}\right)$ vs. X curve given by Table 2-3.

The volumes of the first two CSTRs in series (see Example 2-5) are:

$$\text{Reactor 1:} \quad V_1 = \frac{F_{A0}X_1}{-r_{A1}} \quad (2-13)$$

In this series arrangement, $-r_{A2}$ is evaluated at X_2 for the second CSTR.

$$\text{Reactor 2:} \quad V_2 = \frac{F_{A0}(X_2 - X_1)}{-r_{A2}} \quad (2-24)$$

Starting with the differential form of the PFR design equation

$$F_{A0} \frac{dX}{dV} = -r_A \quad (2-15)$$

rearranging and integrating between limits, when $V = 0$, then $X = X_2$, and when $V = V_3$, then $X = X_3$ we obtain

$$\text{Reactor 3:} \quad V_3 = \int_{X_2}^{X_3} \frac{F_{A0}}{-r_A} dX \quad (2-25)$$

The corresponding reactor volumes for each of the three reactors can be found from the shaded areas in Figure 2-11.

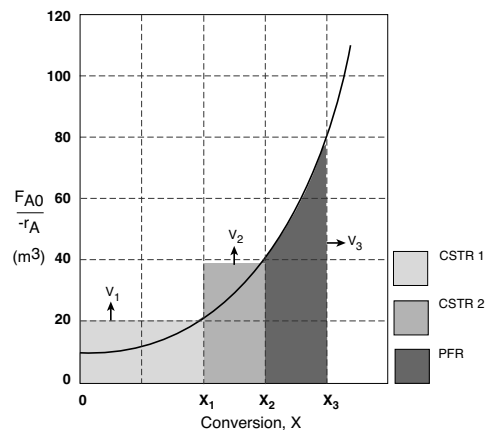
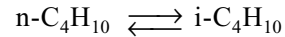


Figure 2-11 Levenspiel plot to determine the reactor volumes V_1 , V_2 , and V_3 .

The $(F_{A0}/-r_A)$ versus X curves we have been using in the previous examples are typical of those found in *isothermal* reaction systems. We will now consider a real reaction system that is carried out *adiabatically*. Isothermal reaction systems are discussed in Chapter 5 and adiabatic systems in Chapter 11.

Example 2-5 An Adiabatic Liquid-Phase Isomerization

The isomerization of butane



was carried out adiabatically in the liquid phase. The data for this reversible reaction are given in Table E2-5.1. (Example 11.3 shows how the data in Table E2-5.1 were generated.)

TABLE E2-5.1 RAW DATA

X	0.0	0.2	0.4	0.6	0.65
$-r_A(\text{kmol}/\text{m}^3 \cdot \text{h})$	39	53	59	38	25

Don't worry how we got this data or why the $(1/-r_A)$ looks the way it does; we will see how to construct this table in Chapter 11, Example 11-3. It is *real data* for a *real reaction* carried out adiabatically, and the reactor scheme shown below in Figure E2-5.1 is used.

Real Data
for a
Real Reaction

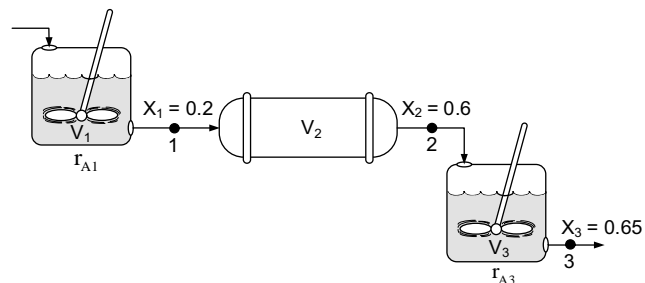


Figure E2-5.1 Reactors in series.

Calculate the volume of each of the reactors for an entering molar flow rate of n-butane of 50 kmol/hr.

Solution

Taking the reciprocal of $-r_A$ and multiplying by F_{A0} , we obtain Table E2-5.2.

$$\text{E.g., at } X = 0: \frac{F_{A0}}{-r_A} = \frac{50 \text{ kmol/h}}{39 \text{ kmol/h} \cdot \text{m}^3} = 1.28 \text{ m}^3$$

TABLE E2-5.2 PROCESSED DATA[†]

X	0.0	0.2	0.4	0.6	0.65
$-r_A (\text{kmol}/\text{m}^3 \cdot \text{h})$	39	53	59	38	25
$[F_{A0}/-r_A] (\text{m}^3)$	1.28	0.94	0.85	1.32	2.0

[†] Data from Table E11-3.1.

(a) For the first CSTR,
when $X = 0.2$, then $\frac{F_{A0}}{-r_A} = 0.94 \text{ m}^3$

$$V_1 = \frac{F_{A0}}{-r_A} X_1 = (0.94 \text{ m}^3)(0.2) = 0.188 \text{ m}^3 \quad (\text{E2-5.1})$$

$$\boxed{V_1 = 0.188 \text{ m}^3 = 188 \text{ dm}^3} \quad (\text{E2-5.2})$$

(b) For the PFR,

$$V_2 = \int_{0.2}^{0.6} \left(\frac{F_{A0}}{-r_A} \right) dX$$

Using Simpson's three-point formula with $\Delta X = (0.6 - 0.2)/2 = 0.2$, and $X_1 = 0.2$, $X_2 = 0.4$, and $X_3 = 0.6$

$$\begin{aligned} V_2 &= \int_{0.2}^{0.6} \frac{F_{A0}}{-r_A} dX = \frac{\Delta X}{3} \left[\left(\frac{F_{A0}}{-r_A} \right)_{X=0.2} + 4 \left(\frac{F_{A0}}{-r_A} \right)_{X=0.4} + \left(\frac{F_{A0}}{-r_A} \right)_{X=0.6} \right] \\ &= \frac{0.2}{3} [0.94 + 4(0.85) + 1.32] \text{m}^3 \end{aligned} \quad (\text{E2-5.3})$$

$$\boxed{V_2 = 0.38 \text{ m}^3 = 380 \text{ dm}^3} \quad (\text{E2-5.4})$$

(c) For the last reactor and the second CSTR, mole balance on A for the CSTR:

In - Out + Generation = 0

$$F_{A2} - F_{A3} + r_{A3} V_3 = 0 \quad (\text{E2-5.5})$$

Rearranging

$$V_3 = \frac{F_{A2} - F_{A3}}{-r_{A3}} \quad (\text{E2-5.6})$$

$$F_{A2} = F_{A0} - F_{A0} X_2$$

$$F_{A3} = F_{A0} - F_{A0} X_3$$

$$V_3 = \frac{(F_{A0} - F_{A0} X_2) - (F_{A0} - F_{A0} X_3)}{-r_{A3}}$$

Simplifying

$$\boxed{V_3 = \left(\frac{F_{A0}}{-r_{A3}} \right) (X_3 - X_2)} \quad (\text{E2-5.7})$$

We find from Table E2-5.2 that at $X_3 = 0.65$, then $\frac{F_{A0}}{-r_{A3}} = 2.0 \text{ m}^3$

$$V_3 = 2 \text{ m}^3 (0.65 - 0.6) = 0.1 \text{ m}^3$$

$$\boxed{V_3 = 0.1 \text{ m}^3 = 100 \text{ dm}^3} \quad (\text{E2-5.8})$$

A Levenspiel plot of $(F_{A0}/-r_A)$ vs. X is shown in Figure E2-5.2.

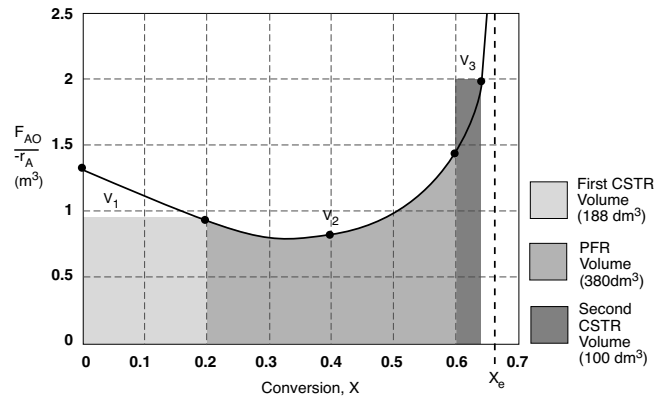


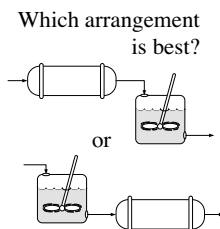
Figure E2-5.2 Levenspiel plot for adiabatic reactors in series.

For this adiabatic reaction the three reactors in series resulted in an overall conversion of 65%. The maximum conversion we can achieve is the equilibrium conversion, which is 68%, and is shown by the dashed line in Figure E2-5.2. Recall that at equilibrium, the rate of reaction is zero and an infinite reactor volume is required to reach equilibrium $\left(V \sim \frac{1}{-r_A} \sim \frac{1}{0} = \infty\right)$.

Analysis: For exothermic reactions that are not carried out isothermally, the rate usually increases at the start of the reaction because reaction temperature increases. However, as the reaction proceeds the rate eventually decreases as the conversion increases as the reactants are consumed. These two competing effects give the bowed shape of the curve in Figure E2-5.2, which will be discussed in detail in Chapter 12. Under these circumstances, we saw that a CSTR will require a smaller volume than a PFR at low conversions.

2.5.4 Comparing the CSTR and PFR Reactor Volumes and Reactor Sequencing

If we look at Figure E2-5.2, the area under the curve (PFR volume) between $X = 0$ and $X = 0.2$, we see that the PFR area is greater than the rectangular area corresponding to the CSTR volume, i.e., $V_{\text{PFR}} > V_{\text{CSTR}}$. However, if we compare the areas under the curve between $X = 0.6$ and $X = 0.65$, we see that the area under the curve (PFR volume) is smaller than the rectangular area corresponding to the CSTR volume, i.e., $V_{\text{CSTR}} > V_{\text{PFR}}$. This result often occurs when the reaction is carried out adiabatically, which is discussed when we look at heat effects in Chapter 11.



In the *sequencing of reactors*, one is often asked, “Which reactor should go first to give the highest overall conversion? Should it be a PFR followed by a CSTR, or two CSTRs, then a PFR, or ...?” The answer is **“It depends.”** It depends not only on the shape of the Levenspiel plot $(F_{A0}/-r_A)$ versus X , but

also on the relative reactor sizes. As an exercise, examine Figure E2-5.2 to learn if there is a better way to arrange the two CSTRs and one PFR. Suppose you were given a Levenspiel plot of $(F_{A0}/-r_A)$ vs. X for three reactors in series, along with their reactor volumes $V_{\text{CSTR1}} = 3 \text{ m}^3$, $V_{\text{CSTR2}} = 2 \text{ m}^3$, and $V_{\text{PFR}} = 1.2 \text{ m}^3$, and were asked to find the highest possible conversion X . What would you do? The methods we used to calculate reactor volumes all apply, except the procedure is reversed and a *trial-and-error solution* is needed to find the exit overall conversion from each reactor (see Problem P2-5B).

Given V and
 $\left[\frac{1}{-r_A} \text{ vs. } X \right]$ find X

The previous examples show that *if* we know the molar flow rate to the reactor and the reaction rate as a function of conversion, *then* we can calculate the reactor volume necessary to achieve a specified conversion. The reaction rate does not depend on conversion alone, however. It is also affected by the initial concentrations of the reactants, the temperature, and the pressure. Consequently, the experimental data obtained in the laboratory and presented in Table 2-1 as $-r_A$ as a function of X are useful only in the design of full-scale reactors that are to be operated at the *identical conditions* as the laboratory experiments (temperature, pressure, and initial reactant concentrations). However, such circumstances are **seldom** encountered and we must revert to the methods we describe in Chapters 3 and 4 to obtain $-r_A$ as a function of X .

Only need
 $-r_A = f(X)$
 to size
 flow reactors

It is important to understand that if the rate of reaction is available or can be obtained solely as a function of conversion, $-r_A = f(X)$, or if it can be generated by some intermediate calculations, one can design a variety of reactors and combinations of reactors.

Chapter 3 shows
 how to find
 $-r_A = f(X)$.

Ordinarily, laboratory data are used to formulate a rate law, and then the reaction rate–conversion functional dependence is determined using the rate law. The preceding sections show that with the reaction rate–conversion relationship, different reactor schemes can readily be sized. In Chapters 3 and 4, we show how we obtain this relationship between reaction rate and conversion from rate law and reaction stoichiometry.

2.6 Some Further Definitions

Before proceeding to Chapter 3, some terms and equations commonly used in reaction engineering need to be defined. We also consider the special case of the plug-flow design equation when the volumetric flow rate is constant.

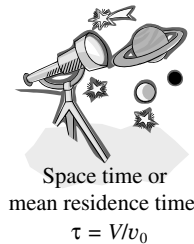
2.6.1 Space Time

The space time tau, τ , is obtained by dividing the reactor volume by the volumetric flow rate entering the reactor

τ is an important
 quantity!

$$\tau \equiv \frac{V}{v_0} \quad (2-26)$$

The space time is the time necessary to process one reactor volume of fluid based on entrance conditions. For example, consider the tubular reactor



shown in Figure 2-12, which is 20 m long and 0.2 m³ in volume. The dashed line in Figure 2-12 represents 0.2 m³ of fluid directly upstream of the reactor. The time it takes for this fluid to enter the reactor completely is called the *space time tau*. It is also called the *holding time* or *mean residence time*.

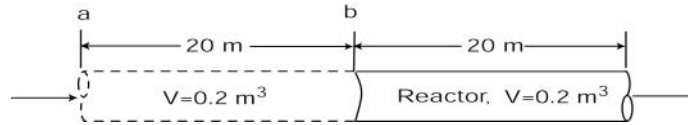


Figure 2-12 Tubular reactor showing identical volume upstream.

For example, if the reactor volume is 0.2 m³ and the inlet volumetric flow rate is 0.01 m³/s, it would take the upstream equivalent reactor volume ($V = 0.2$ m³), shown by the dashed lines, a time τ equal to

$$\tau = \frac{0.2 \text{ m}^3}{0.01 \text{ m}^3/\text{s}} = 20 \text{ s}$$

to enter the reactor ($V = 0.2$ m³). In other words, it would take 20 s for the fluid molecules at point a to move to point b, which corresponds to a space time of 20 s. We can substitute for $F_{A0} = v_0 C_{A0}$ in Equations (2-13) and (2-16) and then divide both sides by v_0 to write our mole balance in the following forms:

$$\text{For a PFR} \quad \tau_p = \left(\frac{V_p}{v_0} \right) = C_{A0} \int_0^X \frac{dX}{-r_A}$$

and

$$\text{For a CSTR} \quad \tau = \left(\frac{V}{v_0} \right) = \frac{C_{A0} X}{-r_A}$$

For plug flow, the space time is equal to the mean residence time in the reactor, t_m (see Chapter 16). This time is the average time the molecules spend in the reactor. A range of typical processing times in terms of the space time (residence time) for industrial reactors is shown in Table 2-4.

TABLE 2-4 TYPICAL SPACE TIME FOR INDUSTRIAL REACTORS²

Practical guidelines	Reactor Type	Mean Residence Time Range	Production Capacity
	Batch	15 min to 20 h	Few kg/day to 100,000 tons/year
CSTR	10 min to 4 h	10 to 3,000,000 tons/year	
Tubular	0.5 s to 1 h	50 to 5,000,000 tons/year	

² Trambouze, Landeghem, and Wauquier, *Chemical Reactors* (Paris: Editions Technip, 1988; Houston: Gulf Publishing Company, 1988), p. 154.

Table 2-5 shows an order of magnitude of the space times for six industrial reactions and associated reactors.

TABLE 2-5 SAMPLE INDUSTRIAL SPACE TIMES³

	<i>Reaction</i>	<i>Reactor</i>	<i>Temperature</i>	<i>Pressure</i> <i>atm</i>	<i>Space Time</i>
(1)	$C_2H_6 \rightarrow C_2H_4 + H_2$	PFR [†]	860°C	2	1 s
(2)	$CH_3CH_2OH + HCH_3COOH \rightarrow$ $CH_3CH_2COOCH_3 + H_2O$	CSTR	100°C	1	2 h
(3)	Catalytic cracking	PBR	490°C	20	1 s < τ < 400 s
(4)	$C_6H_5CH_2CH_3 \rightarrow C_6H_5CH = CH_2 + H_2$	PBR	600°C	1	0.2 s
(5)	$CO + H_2O \rightarrow CO_2 + H_2$	PBR	300°C	26	4.5 s
(6)	$C_6H_6 + HNO_3 \rightarrow$ $C_6H_5NO_2 + H_2O$	CSTR	50°C	1	20 min

Typical industrial
reaction space
times

[†]The reactor is tubular but the flow may or may not be ideal plug flow.

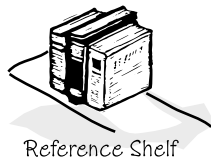
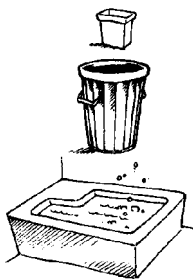


Table 2-6 gives typical sizes for batch and CSTR reactors (along with the comparable size of a familiar object) and the costs associated with those sizes. All reactors are glass lined and the prices include heating/cooling jacket, motor, mixer, and baffles. The reactors can be operated at temperatures between 20 and 450°F, and at pressures up to 100 psi.

TABLE 2-6 REPRESENTATIVE PFAUDLER CSTR/BATCH REACTOR SIZES AND PRICES[†]

<i>Volume</i>	<i>Price</i>
20 dm ³ (Waste Basket)	\$30,000
200 dm ³ (Garbage Can)	\$40,000
2,000 dm ³ (Jacuzzi)	\$75,000
30,000 dm ³ (Gasoline Tanker)	\$300,000

[†]Doesn't include instrumentation costs.



2.6.2 Space Velocity

The space velocity (SV), which is defined as

$$SV \equiv \frac{v_0}{V} \quad SV = \frac{1}{\tau} \quad (2-27)$$

³ Walas, S. M. Chemical Reactor Data, *Chemical Engineering*, 79 (October 14, 1985).

might be regarded at first sight as the reciprocal of the space time. However, there can be a difference in the two quantities' definitions. For the space time, the entering volumetric flow rate is measured at the entrance conditions, but for the space velocity, other conditions are often used. The two space velocities commonly used in industry are the *liquid-hourly* and *gas-hourly space velocities*, LHSV and GHSV, respectively. The entering volumetric flow rate, v_0 , in the LHSV is frequently measured as that of a liquid feed rate at 60°F or 75°F, even though the feed to the reactor may be a vapor at some higher temperature. Strange but true. The gas volumetric flow rate, v_0 , in the GHSV is normally reported at standard temperature and pressure (STP).

$$\text{LHSV} = \frac{v_0|_{\text{liquid}}}{V} \quad (2-28)$$

$$\text{GHSV} = \frac{v_0|_{\text{STP}}}{V} \quad (2-29)$$

Example 2-6 Reactor Space Times and Space Velocities

Calculate the space time, τ , and space velocities for the reactor in Examples 2-1 and 2-3 for an entering volumetric flow rate of 2 dm³/s.

Solution

The entering volumetric flow is 2 dm³/s (0.002 m³/s).

From Example 2-1, the CSTR volume was 6.4 m³ and the corresponding space time, τ , and space velocity, SV are

$$\tau = \frac{V}{v_0} = \frac{6.4 \text{ m}^3}{0.002 \text{ m}^3/\text{s}} = 3200 \text{ s} = 0.89 \text{ h}$$

It takes 0.89 hours to put 6.4 m³ into the reactor.

$$\text{SV} = \frac{1}{\tau} = \frac{1}{0.89 \text{ h}} = 1.125 \text{ h}^{-1}$$

From Example 2-3, the PFR volume was 2.165 m³, and the corresponding space time and space velocity are

$$\tau = \frac{V}{v_0} = \frac{2.165 \text{ m}^3}{0.002 \text{ m}^3/\text{s}} = 1083 \text{ s} = 0.30 \text{ h}$$

$$\text{SV} = \frac{1}{\tau} = \frac{1}{0.30 \text{ h}} = 3.3 \text{ h}^{-1}$$

Analysis: This example gives an *important industrial concept*. These space times are the times for each of the reactors to take the volume of fluid equivalent to one reactor volume and put it into the reactor.

Summary

In these last examples we have seen that in the design of reactors that are to be operated at conditions (e.g., temperature and initial concentration) identical to those at which the reaction rate data were obtained, we can size (determine the reactor volume) both CSTRs and PFRs alone or in various combinations. In principle, it may be possible to scale up a laboratory-bench or pilot-plant reaction system solely from knowledge of $-r_A$ as a function of X or C_A . However, for most reactor systems in industry, a scale-up process cannot be achieved in this manner because knowledge of $-r_A$ solely as a function of X is seldom, if ever, available under identical conditions. By combining the information in Chapters 3 and 4, we shall see how we can obtain $-r_A = f(X)$ from information obtained either in the laboratory or from the literature. This relationship will be developed in a two-step process. In Step 1, we will find the rate law that gives the rate as a function of concentration (Chapter 3) and in Step 2, we will find the concentrations as a function of conversion (Chapter 4). Combining Steps 1 and 2 in Chapters 3 and 4, we obtain $-r_A = f(X)$. We can then use the methods developed in this chapter, along with integral and numerical methods, to size reactors.

Coming attractions
in Chapters 3 and 4

The CRE Algorithm

- Mole Balance, Ch 1
- Rate Law, Ch 3
- Stoichiometry, Ch 4
- Combine, Ch 5
- Evaluate, Ch 5
- Energy Balance, Ch 11

Closure

In this chapter, we have shown that if you are given the rate of reaction as a function of conversion, i.e., $-r_A = f(X)$, you will be able to size CSTRs and PFRs, and arrange the order of a given set of reactors to determine the maximum overall conversion. After completing this chapter, the reader should be able to

- a. Define the parameter *conversion* and rewrite the mole balances in terms of conversion
- b. Show that by expressing $-r_A$ as a function of conversion X , a number of reactors and reaction systems can be sized or a conversion calculated from a given reactor size
- c. Arrange reactors in series to achieve the maximum conversion for a given Levenspiel plot

SUMMARY

1. The conversion X is the moles of A reacted per mole of A fed.

$$\text{For batch systems:} \quad X = \frac{N_{A0} - N_A}{N_{A0}} \quad (\text{S2-1})$$

$$\text{For flow systems:} \quad X = \frac{F_{A0} - F_A}{F_{A0}} \quad (\text{S2-2})$$

For reactors in series with no side streams, the conversion at point i is

$$X_i = \frac{\text{Total moles of A reacted up to point } i}{\text{Moles A fed to the first reactor}} \quad (\text{S2-3})$$

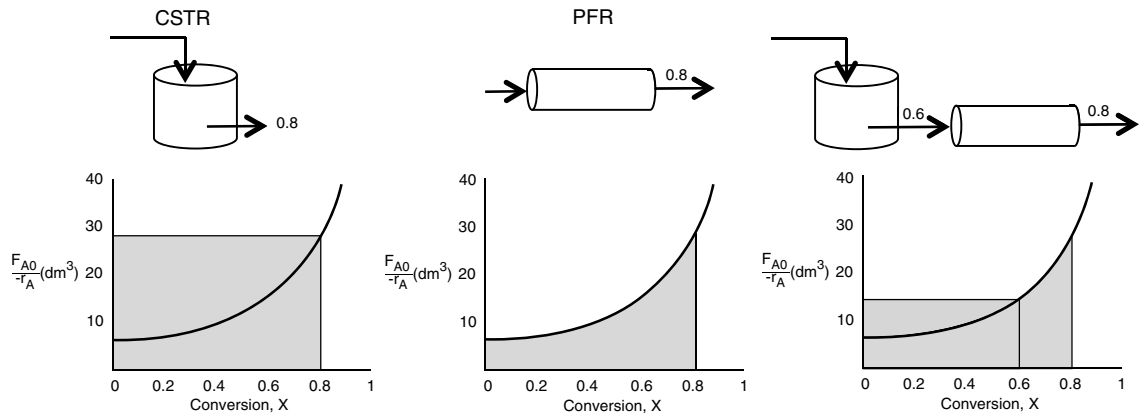
2. In terms of the conversion, the differential and integral forms of the reactor design equations become:

TABLE S2-1 MOLE BALANCE FOR BR, CSTR, PFR, AND PBR TERMS OF CONVERSION

	Differential Form	Algebraic Form	Integral Form
Batch	$N_{A0} \frac{dX}{dt} = -r_A V$		$t = N_{A0} \int_0^X \frac{dX}{-r_A V}$
CSTR		$V = \frac{F_{A0}(X_{out} - X_{in})}{(-r_A)_{out}}$	
Fluidized CSTR		$W = \frac{F_{A0}(X_{out} - X_{in})}{(-r'_A)_{out}}$	
PFR	$F_{A0} \frac{dX}{dV} = -r_A$		$V = F_{A0} \int_{X_{in}}^{X_{out}} \frac{dX}{-r_A}$
PBR	$F_{A0} \frac{dX}{dW} = -r'_A$		$W = F_{A0} \int_{X_{in}}^{X_{out}} \frac{dX}{-r'_A}$

3. If the rate of disappearance of A is given as a function of conversion, the following graphical techniques can be used to size a CSTR and a plug-flow reactor.

A. Graphical Integration Using Levenspiel Plots



The PFR integral could also be evaluated by

B. Numerical Integration

See Appendix A.4 for quadrature formulas such as the five-point quadrature formula with $\Delta X = 0.8/4$ of five equally spaced points, $X_1 = 0$, $X_2 = 0.2$, $X_3 = 0.4$, $X_4 = 0.6$, and $X_5 = 0.8$.

4. Space time, τ , and space velocity, SV, are given by

$$\tau = \frac{V}{v_0} \quad (\text{S2-4})$$

$$\text{SV} = \frac{v_0}{V} \quad (\text{at STP}) \quad (\text{S2-5})$$

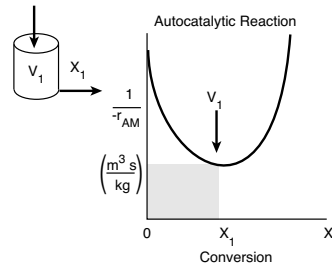
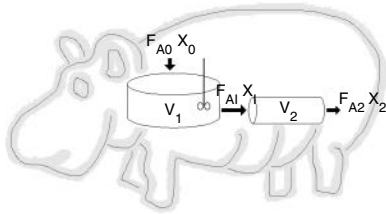
CRE WEB SITE

- **Expanded Material**

1. *Web P2-1_A Reactor Sizing for Reversible Reactions*
2. *Web P2-2_A Puzzle Problem “What’s Wrong with this Solution?”*

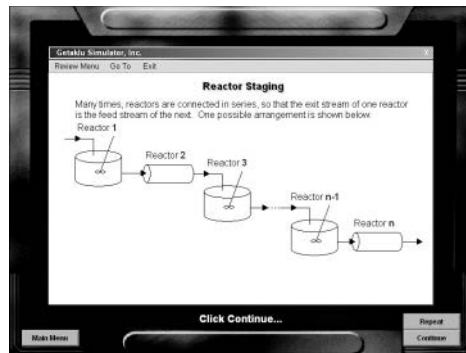
- **Learning Resources**

1. *Summary Notes for Chapter 2*
2. *Web Module*
Hippopotamus Digestive System



Levenspiel Plot for Autocatalytic Digestion in a CSTR

3. *Interactive Computer Games*
Reactor Staging



4. *Solved Problems*

A. CDP2-A_B More CSTR and PFR Calculations—No Memorization

- **FAQ (Frequently Asked Questions)**
- **Professional Reference Shelf**

QUESTIONS AND PROBLEMS

The subscript to each of the problem numbers indicates the level of difficulty: A, least difficult; D, most difficult.

A = ● B = ■ C = ◆ D = ◆◆

Questions

- Q2-1** (a) Without referring back, make a list of the most important items you learned in this chapter.
 (b) What do you believe was the overall purpose of the chapter?
 Before solving the problems, state or sketch qualitatively the expected results or trends.
- Q2-2** Go to the Web site www.engr.ncsu.edu/learningstyles/ilsweb.html.
 (a) Take the Inventory of Learning Style test, and record your learning style according to the Solomon/Felder inventory.
 Global/Sequential _____
 Active/Reflective _____
 Visual/Verbal _____
 Sensing/Intuitive _____
 (b) After checking the CRE Web site, www.umich.edu/~elements/asyLearn/learningstyles.htm, suggest two ways to facilitate your learning style in each of the four categories.

Problems

- P2-1_A** (a) Revisit **Examples 2-1** through **2-3**. How would your answers change if the flow rate, F_{A0} , were cut in half? If it were doubled? What conversion can be achieved in a 4.5 m³ PFR and in a 4.5 m³ CSTR?
 (b) Revisit **Example 2-2**. Being a company about to go bankrupt, you can only afford a 2.5 m³ CSTR. What conversion can you achieve?
 (c) Revisit **Example 2-3**. What conversion could you achieve if you could convince your boss, Dr. Pennypincher, to spend more money to buy a 1.0 m³ PFR to attach to a 2.40 CSTR?
 (d) Revisit **Example 2-4**. How would your answers change if the two CSTRs (one 0.82 m³ and the other 3.2 m³) were placed in parallel with the flow, F_{A0} , divided equally between the reactors.
 (e) Revisit **Example 2-5**. (1) What would be the reactor volumes if the two intermediate conversions were changed to 20% and 50%, respectively? (2) What would be the conversions, X_1 , X_2 , and X_3 , if all the reactors had the same volume of 100 dm³ and were placed in the same order? (3) What is the worst possible way to arrange the two CSTRs and one PFR?
 (f) Revisit **Example 2-6**. If the term $C_{A0} \int_0^X \frac{dX}{-r_A}$ is 2 seconds for 80% conversion, how much fluid (m³/min) can you process in a 3 m³ reactor?
- P2-2_A** **ICG Staging**. Download the Interactive Computer Game (ICG) from the CRE Web site. Play this game and then record your performance number, which indicates your mastery of the material. Your professor has the key to decode your performance number. Note: To play this game you *must* have Windows 2000 or a later version.
 ICG Reactor Staging Performance # _____
- P2-3_B** You have two CSTRs and two PFRs, each with a volume of 1.6 m³. Use Figure 2-2B on page 41 to calculate the conversion for each of the reactors in the following arrangements.
 (a) Two CSTRs in series. (*Ans.*: $X_1 = 0.435$, $X_2 = 0.66$)
 (b) Two PFRs in series.
 (c) Two CSTRs in parallel with the feed, F_{A0} , divided equally between the two reactors.
 (d) Two PFRs in parallel with the feed divided equally between the two reactors.

